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Ranges and Paths

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Abstract Range expressions such as *between 3 and 8*, *from 3 to 8*, and *3 through 8* resemble modified numerals such as *at least 3* and have sometimes been mentioned under that rubric. This paper shows that they are crucially different in their distribution, the readings available to them, and their behavior with respect to quantifiers, and more generally that they have an intricate grammar of their own. We distinguish three classes of readings they can receive: singleton punctual readings, on which they often give rise to ignorance inferences; set punctual readings, which arise chiefly in the scope of quantifiers; and interval readings, where the range is interpreted exhaustively. The heart of the semantics of range expressions, we suggest, is the notion of paths, generalized from its use in the analysis of locatives to extend across semantic types.

Keywords ranges, modified numerals, measure phrases, locatives, paths, indefinites, ignorance inferences

1 Introduction

This paper will take up the challenge of range expressions such as those in (1):

(1) a. Floyd saw <u>between 10 and 15</u> ferrets.
b. The kids' ages ranged <u>from 10 to 15</u>.

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c. This volume covers $\left\{\begin{array}{l}\underline{\text{from Lincoln to Taft}}\\ \underline{\text{Lincoln through Taft}}\end{array}\right\}$.

These expressions are akin to modified numerals such as *at least 3*, but as we'll show, they have an interesting and subtle grammar of their own. That's especially clear in (1c), which shows that a range needn't be numeric. Any set with an appropriate ordering—such as the alphabet or American presidents—will suffice.[1]

We take range expressions to be, structurally, a special case of a more general phenomenon that in other work (Gobeski & Morzycki (2022a)) we've termed COMPOSITE MEASURE PHRASES:

(2) a. The room is 10 feet by 15 feet.

b. Her odds of winning are $\left\{\begin{array}{l}\text{1 in 4}\\ \text{1 to 3}\\ \text{1 out of 4}\end{array}\right\}$.

c. The Padres beat the Red Sox by 3 to 2.

The distinguishing characteristic of these is that they're measure phrases with multiple measure phrases as subconstituents. The resemblance is chiefly syntactic, and we won't linger on it further here, but it's relevant to the larger point that these more complicated structural arrangements have subtle and intriguing semantic consequences.

Range expressions have sometimes been brought up as a form of modified numeral (Nouwen 2010, Rett 2014a, Buccola & Spector 2016, Schwarz et al. 2012), but there is a considerably larger story to be told. That's reflected most clearly in non-numeric examples like (1c), but also in the selectional restrictions of predicates, among other things.

Section 2 lays out the crucial empirical facts, identifying a diagnostic for range expressions that distinguishes them from ordinary modified numerals. It also divides the readings available to them into three classes, and provides a diagnostic that adjudicates among them. Section 3 addresses the locative foundations on which the grammar of range expressions rests, and introduces the crucial tool, paths, that will serve to capture the essential core of their semantics. Section 4 takes up the broader challenge of range expressions in earnest. It provides a semantics for their in individual, degree, and cardinality contexts by generalizing the notion of paths across semantic types. Along the

[1]We won't pursue here the question of what properties this ordering must have, but it apparently needn't actually be a true partial order at all. American presidents have served non-concurrent terms, but it's natural to build ranges based on temporal precedence. That entails that the presidential temporal precedence relation is not antisymmetric, making it only a preorder.

way, evidence emerges in this section for encoding maximality in a particular place in the semantics of adjectives. Section 5 concludes.

2 The data

2.1 *How range expressions are built*

Cast of characters There are at least three basic varieties of range expressions in English, each introduced by distinct morphemes:

(3) a. between 3 and 8
b. (from) 3 to 8
c. 3 through 8

It is possible to combine some of these basic types into hybrid forms, as in *from 3 through to 8*. There may also be a fourth type, which expresses ranges bounded on only one end:

(4) a. from 3 up
b. up to 8

In both the cases in (4), *up* is crucial to building a range expression. Of course, *from 3* and *to 8* are in principle grammatical, but they don't occur in range expressions:

(5) a. the range from 3 up
b. *the range from 3

(6) a. The kids ages were from 3 up.
b. *The kids ages were from 3.

The fact that these can be combined (*from 3 up to 8*) and that they resemble (3b) suggests that the *up* version may be a variant of (3b) rather than a distinct form. *Up to* is discussed extensively in Schwarz et al. (2012).

Predicates that select ranges First, as noted in section 1, there is a class of nouns and verbs that select range complements exclusively. This includes both the nominal and verbal forms of *range* and *span*, as well as verbs *cover*:

(7) a. $\text{the} \left\{\begin{array}{l}\text{range}\\ \text{span}\end{array}\right\} \left\{\begin{array}{l}\text{ between 2 and 6}\\ \text{ from 2 to 6}\\ \text{ 2 through 6}\\ \text{ up to 6}\\ {}^{\#}\text{at least 2}\\ {}^{\#}\text{more than 2}\\ {}^{\#}\text{around 4}\\ {}^{\#}\text{under 6}\end{array}\right\}$

b. $\text{This volume covers} \left\{\begin{array}{l}\text{ between G and L}\\ \text{ from G to L}\\ \text{ G through L}\\ \text{ up to L}\\ {}^{\#}\text{at least G}\\ {}^{\#}\text{more than G}\\ {}^{\#}\text{around G}\\ {}^{\#}\text{under L}\end{array}\right\}.$

c. $\text{Their ages} \left\{\begin{array}{l}\text{ranged}\\ \text{spanned}\\ \text{varied}\end{array}\right\} \left\{\begin{array}{l}\text{ between 2 and 6}\\ \text{ from 2 to 6}\\ \text{ 2 through 6}\\ \text{ up to 6}\\ {}^{\#}\text{at least 2}\\ {}^{\#}\text{more than 2}\\ {}^{\#}\text{around 4}\\ {}^{\#}\text{under 6}\end{array}\right\}.$

As (7) reflects, typical modified numerals such as *at least 2* are impossible in these positions. That's an initial important indication that it's fruitful to distinguish range expressions from arbitrary cases of numeral modification. For this reason, this is a natural diagnostic for range expressions, especially in contradistinction to (other) modified numerals.

Predicates of ranges Some range expressions can be predicated of a particular number or, more generally, a particular element in an order. The prime example of this is *between/and*, which contrasts with *from/to* and *through* in this respect:

(8) a. $\text{3 is} \left\{\begin{array}{l}\text{ between 1 and 5}\\ {}^{\#}\text{from 1 to 5}\\ {}^{\#}\text{1 through 5}\end{array}\right\}.$

b. $\text{Taft is} \left\{\begin{array}{l}\text{ between Lincoln and Eisenhower}\\ {}^{\#}\text{from Lincoln to Eisenhower}\\ {}^{\#}\text{Lincoln through Eisenhower}\end{array}\right\}.$

c. Montreal is { between Vancouver and Toronto / #from Vancouver to Toronto / #Vancouver through Toronto }.

Ranges built with *from/to* require an extended portion of an order, that is, an interval across time, space, degrees, or numbers:

(9) a. The meeting was from 11:00 to 12:00.
b. The trip was from Vancouver to Toronto.
c. This box is from 8 to 9 inches wide.
d. The possible values of this variable range from 8 to 9.
e. This line is from point A to point B.
f. We saw from 5 to 10 capybaras.

There are some important further distinctions here, which we will encounter subsequently. One worth noting immediately: in (9c) and (9f), the range of values is created by uncertainty around where a particular value is located. (We address this in 2.4.) *Through* can grammatically be inserted in the examples in (9), but bare *through* range expressions are not in general grammatical in these contexts:

(10) a. The meeting was 11:00 through 12:00.
b. ??The trip was from Vancouver through Toronto.[2]
c. #This box is 8 through 9 inches wide.
d. #The possible values of this variable range 8 through 9.
e. #This line is point A through point B.
f. #We saw from 5 through 10 capybaras.

On the other hand, *between* is possible in all of them:

(11) a. The meeting was between 11:00 and 12:00.
b. The trip was between Vancouver and Toronto.
c. This box is between 8 and 9 inches wide.
d. The possible values of this variable range between 8 and 9.
e. This line is between point A and point B.
f. We saw between 5 and 10 capybaras.

The upshot, then, is that *from/to* ranges are fundamentally predicates of ranges themselves, whereas *between* can be predicated both of a range of values and of a single one. It's worth noting as well that this distinction emerges in the event domain, where it has aspectual consequences. *From/to* ranges are generally odd with telic predicates:

[2] This seems to be possible for airline employees.

(12) a. He pushed the cart $\left\{\begin{array}{l}\text{between here and Toronto}\\ \text{from here to Toronto}\\ \text{from here through to Toronto}\end{array}\right\}$.

b. He exploded $\left\{\begin{array}{l}\ \ \text{between here and Toronto}\\ {}^{\#}\text{from here to Toronto}\\ {}^{\#}\text{from here through to Toronto}\end{array}\right\}$.

Orders provided by context The interpretation of a range expression is sometimes constrained by a contextually-supplied order that contains its range. For example, an apparently exhaustive range expression like *houses 10 through 15* may pick out a range of houses that excludes 13 if that number has been skipped by city planners for superstitious reasons, and of course it would normally (but not necessarily!) skip fractional house numbers as well. This would be different if the context were different. In a discussion that makes the integers the salient order, skipping 13 would not be possible, but skipping fractions still would. This contextual effect is examined in more detail in section 3.3.

Distinctness and non-zero requirements Range expressions also require that the two constituents be distinct from each other:

(13) a. #Sam saw between 3 and 3 monkeys.
b. #Floyd ate a dozen to 12 cookies.

This is unsurprising, of course, but neither is it trivial. The requirement holds even when the numbers are introduced under different descriptions:

(14) #between 3 and the number on this playing card *(odd if both are 3)*

Nor, under typical circumstances, can ranges include zero except on a jokey reading:

(15) a. #?Sam saw between 0 and 3 monkeys.
b. #?We expect 0 to 3 people.
c. Floyd is $\left\{\begin{array}{c}{}^{\#}0\\ 1\end{array}\right\}$ to 6 feet tall.

This restriction is present only where *0* is either independently infelicitous (#*0 feet tall*) or a receives a reading similar to “no” (*Sam saw 0 monkeys*; Bylinina & Nouwen 2018). Environments in which *0* is felicitous also permit ranges that contain *0*:

(16) a. This car goes from 0 to 60 (miles per hour) in 8 seconds.
b. This toy is for ages 0 through 3.

These facts in conjunction are likely the same phenomenon as Schwarz et al. (2012)'s BOTTOM-OF-SCALE EFFECTS. They observe that an expression such as *up to* doesn't normally include 0 in the range:

(17) #Up to 1 person died in the crash.

The measurement in (17) starts at 1—and since the two range expressions must be distinct, (17) is infelicitous.

Linear order restriction The linear order of the constituents of a range expression is required to accord with the order on the underlying scale:

(18) a. Sam saw $\left\{\begin{array}{l}\text{between 3 and 6}\\ \text{\#between 6 and 3}\end{array}\right\}$ monkeys.

b. We expect $\left\{\begin{array}{l}\text{(from) 3 to 6}\\ \text{\#(from) 6 to 3}\end{array}\right\}$ people.

c. This class is for ages $\left\{\begin{array}{l}\text{3 through 6}\\ \text{\#6 through 3}\end{array}\right\}$.

This restriction is present for non-numeric ranges, as well as for negative adjectives such as *short*. That's important because, on some theories of polar antonyms, the scale of a negative adjective is taken to be the same as for a positive adjective but with the ordering inverted (see, among others, Kennedy 2001, Heim 2006). Thus one might expect order of elements in a range expression to invert as well:

(19) Floyd is $\left\{\begin{array}{l}\text{4 to 7}\\ \text{\#7 to 4}\end{array}\right\}$ inches shorter than Clyde.

In this respect, range expressions may provide a useful proving ground for theories of scale structure and polar antonyms.

Constituency facts There is room for skepticism about whether *from/to* range expressions are actually constituents at all, rather than merely a common collocation involving distinct adjacent constituents. One might suspect them of being simply adjacent locative PP adjuncts. But the order of truly locative PPs can be inverted:

(20) Floyd drove $\left\{\begin{array}{l}\text{from Boston to Cleveland}\\ \text{from Cleveland to Boston}\\ \text{between Boston and Cleveland}\\ \text{between Cleveland and Boston}\end{array}\right\}$. *(locative)*

That is not the case for range expressions:

(21) We expect { from 3 to 6 / *to 6 from 3 } people. *(range)*

Similarly, locative PPs cleft, but components of ranges don't:

(22) a. It was to Cleveland that we drove from Boston __. *(locative)*
b. *It was to 6 that we expect from 3 __ people. *(range)*

The analysis pursued here will indeed draw a close connection between range and locative expressions, but crucially, it will not have the consequence that they are identical. More important still, part of the interest will be in how range expressions acquire an intricate variety of distinct uses by building on their locative foundations.

2.2 *Distribution*

The previous section examined the internal structure of range expressions and some of the essential semantic distinctions associated with it. The task in this section will be to briefly note the diversity of syntactic contexts in which range expressions occur. We've already encountered examples of range expressions in predicative positions, such as *4 is between 2 and 6*, as well as in argument position, such as the complement of both the verb and the noun *range*, as in *Possible values range from 2 to 6* or *the range from 2 to 6*. Two other syntactic contexts bear highlighting.

Measure phrases Range expressions have uses as measure phrases in both adjectival and prepositional environments:

(23) a. Floyd is (from) 5 to 6 feet tall.
b. Winnipeg is (from) 500 to 600km away from most things.

Adjuncts They are also possible as adjuncts:

(24) a. The presidents from Lincoln to Taft were in favor of facial hair.
b. Think of any number between 6 and 20.
c. Children { from 6 to 10 / between 6 and 10 / 6 through 10 } are permitted.

There are some puzzling contrasts distinguishing predicative and adjunct uses:

(25) a. #Each kid is from 5 to 10. *(predicative)*
b. Each kid from 5 to 10 participated. *(adjoined)*
c. #?Each kid who is from 5 to 10 participated. *(predicative in a relative clause)*

2.3 *Anaphora*

Anaphora to ranges is possible:

(26) Floyd saw 5 to 10 million bees and Clyde saw that many bees too.

Crucially, *that many* can refer to the full range here, not just to one of its subcomponents, so Clyde and Floyd may have seen different numbers of bees. Anaphora is also possible via degree nominalizations:

(27) This group is for kids 10 through 14, and that one is for kids that age too.

2.4 *Three kinds of readings*

We'll distinguish three kinds of readings that range expressions can receive, depending on their environment. We dub these SINGLETON PUNCTUAL READINGS, INTERVAL READINGS, and SET PUNCTUAL READINGS.[3]

Singleton punctual readings The reading that comes most readily to mind is the singleton punctual reading, on which there is a single crucial value within the range, which the speaker may or may not know:

(28) a. Floyd is $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\end{array}\right\}$ feet tall.

b. Floyd weighs $\left\{\begin{array}{l}\text{between 150 and 200}\\ \text{from 150 to 200}\end{array}\right\}$ pounds.

c. We expect $\left\{\begin{array}{l}\text{between 10 and 15}\\ \text{from 10 to 15}\end{array}\right\}$ people.

In (28a), for example, the value of interest is normally Floyd's (maximum) degree of height. The range itself is present only as an epistemic fig leaf covering the speaker's uncertainty.

[3]The use of 'punctual' in this empirical neighborhood is inspired by Rett (2014a).

Interval readings By contrast, on interval readings the whole of the range is crucial:

(29) a. Water is fluid $\left\{\begin{array}{l}\text{between 32 and 212}\\ \text{from 32 to 212}\\ \text{at 32 through 212}\end{array}\right\}$ degrees Fahrenheit.

b. This volume covers American presidents $\left\{\begin{array}{l}\text{between Lincoln and Taft}\\ \text{from Lincoln to Taft}\\ \text{Lincoln through Taft}\end{array}\right\}$.

c. The snow plow cleared $\left\{\begin{array}{l}\text{between 12 and 20 Main St.}\\ \text{from 12 to 20 Main St.}\end{array}\right\}$

In (29a), we're not interested in a single temperature at which water is fluid, but rather the full interval of temperatures. Importantly, *through* seems to receive only interval readings. There is a helpful diagnostic for these readings—they alone allow exceptives:

(30) a. Water is fluid $\left\{\begin{array}{l}\text{between 32 and 212}\\ \text{from 32 to 212}\\ \text{at 32 through 212}\end{array}\right\}$ degrees Fahrenheit, **except at 100 degrees, for some reason.**

b. This volume covers American presidents $\left\{\begin{array}{l}\text{between Lincoln and Taft}\\ \text{from Lincoln to Taft}\\ \text{Lincoln through Taft}\end{array}\right\}$ **except (for) Garfield.**

c. The snow plow cleared $\left\{\begin{array}{l}\text{between 12 and 20 Main St.}\\ \text{from 12 to 20 Main St.}\end{array}\right\}$ **except for 16 Main St.**

Of course, (30a) is false, but it is perfectly well-formed, as is (30b). Range expressions that receive a singleton punctual reading don't support exceptives:

(31) a. Floyd is $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\end{array}\right\}$ feet tall $^{\#}$**except 5 foot 8.**

b. Floyd weighs $\left\{\begin{array}{l}\text{between 150 and 200}\\ \text{from 150 to 200}\end{array}\right\}$ pounds $^{\#}$**except 175.**

c. We expect $\left\{\begin{array}{l}\text{between 10 and 15}\\ \text{from 10 to 15}\end{array}\right\}$ people $^{\#}$**except 12.**

Set punctual readings On these readings, there are potentially multiple values in the range that are crucial—though unlike on interval readings, these crucial values needn't exhaust the full range. Set punctual readings arise mainly in plural and quantified contexts (including modals):

(32) a. The children were $\left\{\begin{array}{l}\text{(from) 5 to 6}\\ \text{between 5 and 6}\end{array}\right\}$ feet tall. *(plural)*

b. Every student was $\left\{\begin{array}{l}\text{5 to 6}\\ \text{between 5 and 6}\\ {}^{?}\text{from 5 to 6}\end{array}\right\}$ feet tall. *(quantificational)*

c. To ride the roller coaster,
you must be $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\\ {}^{?}\text{from 5 to 6}\end{array}\right\}$ feet tall. *(modal)*

They are also possible in temporal examples such as (33):

(33) Floyd's weight fluctuated $\left\{\begin{array}{l}\text{between 175 and 200}\\ \text{from 175 to 200}\end{array}\right\}$ pounds. *(temporal)*

These readings resemble interval readings in that they involve more than one crucial value, but the exceptive diagnostic demonstrates that they are in fact distinct—exceptives are very odd with set punctual range expressions:

(34) a. The children were $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{(from) 5 to 6}\end{array}\right\}$ feet tall, $^{??}$**except (for) 5 foot 3.**

b. Every student was $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\end{array}\right\}$ feet tall, $^{??}$**except (for) 5 foot 3.**

c. To ride this roller coaster, you must be $\left\{\begin{array}{l}\text{between 5 and 6}\\ \text{5 to 6}\end{array}\right\}$ feet tall, $^{??}$**except (for) 5 foot 3.**

The quantificational and plural forms in (34a) and (34b), respectively, are odd at best, while the modal form in (34c) is truly strange.

3 Laying the analytical groundwork: the locative foundations of range expressions

3.1 *From locatives to ranges*

We will begin our analysis in what may seem a surprising place: locative adverbials.[4] As we demonstrated in 2.1, range expressions are actually fundamentally different from superficially similar locative expressions. In English, this is most clearly demonstrated by the fact that range expressions have an inherent two-part syntactic structure with elements that can't be inverted—*from 5 to 10* is possible, but not **to 10 from 5*—which distinguishes them from two adjacent locative PPs.

Nevertheless, it's unmistakable that there is *some* connection between range expressions and locatives, so it doesn't seem too bold to suppose that ranges are built on the grammatical scaffolding of locatives.[5] An alternative starting point would be the connection to modified numerals, which is in fact a course we pursued in earlier work (Gobeski & Morzycki (2022b)). We'll return to the issue of what the relative benefits of these two approaches are, but for now let's focus on the locative one.

Our first step will be to commit to some representational assumptions about locative expressions. Following Zwarts (2005) directly—and, less directly, a line of research in Vector Space Semantics (Zwarts 1997, Zwarts & Winter 2000) and the approach to degree semantics in Schwarzschild (2012, 2013)—we will assume a model that includes points in space as well as *paths* in space. A path is more or less what it sounds like: a sequence of adjacent points with a direction, going from a start point to an end point. The type of paths is p, and we'll use the variables p, p', p'', and so on. There is a wide range of additional decisions to be made about the nature of paths empirically and the formal properties of their representations. These include whether paths have to be, informally, straight lines in space that cover the shortest distance in space between a start point and an end point. The alternative would be to suppose that they can contain turns and curves. Another decision concerns whether paths can, intuitively, 'skip over' a few points and whether they must be dense, in the way that natural language scales generally have been claimed to be (Fox & Hackl 2006). These are reasonable questions, but we will defer them for now because resolving them isn't crucial to getting started, and it will be more profitable to return to them later with more

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[4]For convenience, we henceforth will use 'locative' to refer to expressions that are locative in a narrow sense like *above the house* as well as directional expressions like *from the house* (Zwarts 2005 for more on the distinction and Zwarts 2017 for a recent more general appraisal of locative semantics).

[5]Hackl (2023) helpfully nudged us in this direction.

assumptions in place. We will do this in section 4.1, where the notion of paths will be generalized crosscategorially.

To build denotations, we will appeal to functions that retrieve from a path its start point and its end point: **start** and **end**, again building on Zwarts (1997), Zwarts & Winter (2000), Schwarzschild (2012, 2013), Faller (1998), Zwarts & Winter (2000). These are essentially like Neo-Davidsonian thematic role predicates such as **agent** that retrieve a particular kind of participant from an event (Parsons 1990 among many others).

With this in place, we can treat locative PP like *from Los Angeles* and *to New York* as a properties of paths that, respectively, start at Chicago and end at New York:

(35) $[\![\textit{from Los Angeles}]\!] = \lambda p[\textbf{start}(p) = \textbf{Los-Angeles}]$
$[\![\textit{to New York}]\!] = \lambda p[\textbf{end}(p) = \textbf{New-York}]$

This brings to light another formal decision. It makes sense to think of the start point of a path as, well, a point, rather than an ordinary individual like Los Angeles, as (35) would seem to suggest. We could, of course, reflect that in the denotation, writing **location**(**Los-Angeles**) instead of just **Los-Angeles** to refer to the point at which Los Angeles is located, and likewise throughout:

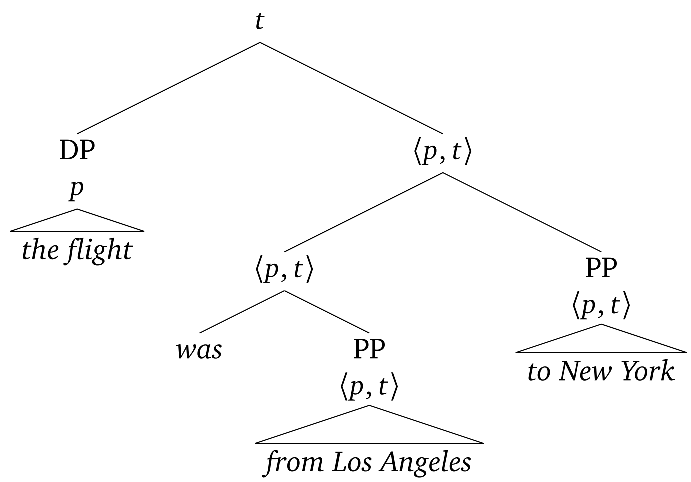
(36) $[\![\textit{from Los Angeles}]\!] = \lambda p[\textbf{start}(p) = \textbf{location}(\textbf{Los-Angeles})]$
$[\![\textit{to New York}]\!] = \lambda p[\textbf{end}(p) = \textbf{location}(\textbf{New-York})]$

But for simplicity, we'll adopt the convention that, as in (35), terms for individuals can be used to refer instead to the point at which those individuals are located.[6]

To interpret a simple locative sentence, we will need to further assume that certain nominals can refer to paths. Among them are *trip*, *flight*, *journey*, and *path* itself. A simple locative sentence could thus be interpreted as in (37):

[6] This sets aside interesting questions of vagueness associated with whether Los Angeles can actually be said to be located at a point rather than a region of points and where its boundaries lie.

(37)



a. $[\![$ *was from Los Angeles* $]\!] = [\![$ *from Los Angeles* $]\!]$
$= \lambda p[\mathbf{start}(p) = \mathbf{Los\text{-}Angeles}]$

b. $[\![$ *was from Los Angeles to New York* $]\!]$
$= \lambda p[\mathbf{start}(p) = \mathbf{Los\text{-}Angeles} \wedge \mathbf{end}(p) = \mathbf{New\text{-}York}]$

c. $[\![$ *the flight was from Los Angeles to New York* $]\!]$
$= 1$ iff $\mathbf{start}(\mathbf{the\text{-}flight}) = \mathbf{Los\text{-}Angeles} \wedge \mathbf{end}(\mathbf{the\text{-}flight}) = \mathbf{New\text{-}York}$

We take *was* to be vacuous here. We've treated the *from* and *to* PPs here as separate constituents, with the *to* PP being an adjunct interpreted intersectively. That seems to be appropriate for locative uses, but not in range expressions, as section 2.1 noted.

One might reasonably be skeptical about whether an eventive nominalization like *the flight* actually refers to a path. A flight can be enjoyable or boring or comfortable, but none of these could sensibly be predicated of just a set of points. Really, we should say that an individual like a flight or a trip has a PATH CORRELATE, and it's that path correlate that can be said to have a start or end point. For the moment, it will suffice to suppose that a type shift from individuals to their path correlates (type $\langle e, p \rangle$) can freely apply anywhere, and in this case it applied to the subject. An alternative course would be to build this shift into the denotation of the range expression. Yet a third option would be to suppose the predicates **start** and **end** actually map events directly to the location where they start and end.

The semantics in (37) makes a prediction worth noting now because it will prove important. Nominals that don't denote or quantify over paths (whether inherently or as the result of a shift) cannot occur in the subject position of sentences like (37):

(38) a. $^{\#}$Chicago is from Los Angeles to New York.
b. $^{\#}$Chicago is to New York.

There is a wrinkle here, though. Plurals can sometimes be shifted to path interpretations, subject to mysterious restrictions:

(39) The strikes were $\left\{\begin{array}{l} \text{ from Cleveland to Cincinnati} \\ {}^{\#}\text{from Cleveland} \\ {}^{\#}\text{to Cincinnati} \end{array}\right\}$.

It would be possible to accommodate such uses on our approach with the aid of a distributivity operator that universally quantifies over atomic strikes and locates each of them along a path from Cleveland to Cincinnati, but that wouldn't address why the non-path subject is possible here at all.

3.2 *Between two points*

Our current aim is to sketch a portion of the grammar of the locative prepositions that will be crucial ingredients in building range expressions across categories. The ingredient is *between*. We'll approach it in stages.

First, *between* PPs differ from *from/to* PPs in that *between* is not normally a predicate of paths. Thus the ungrammatical examples with non-path subjects in (38) are natural with *between*:

(40) Chicago is between Los Angeles and New York.

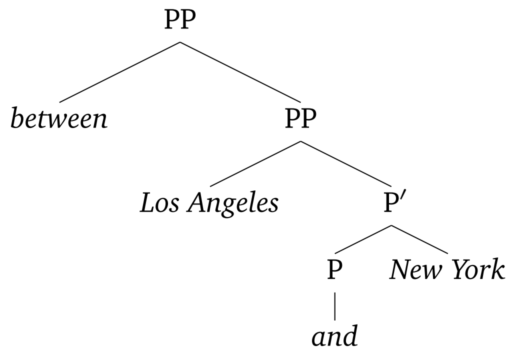
There are path uses alongside this, though (as already noted, in different terms, in section 2.1):

(41) The flight is between Los Angeles and New York.

In both uses, *between* specifies two endpoints.

Second, in order to assemble the pieces, it makes sense for the moment to construe *between* as actually having *two* internal arguments, one for each path endpoint. The natural alternative would be a single internal argument that's plural, but as we will see this is only feasible for locative cases. The syntax required to achieve a structure with two internal arguments isn't straightforward and our focus here is semantic, but one might think of *between/and* as single discontinuous preposition. This could be implemented by supposing that it is a kind of idiom, with gaps for the two arguments, one of them in a specifier position of an embedded PP:

(42)



Alternatively, one might derive this structure by placing *between/and* in the internal P position and raising *between* by head movement to the higher P.

With that, we can venture a denotation, one on which a *between* PP holds of an individual iff it is located on a path between the two endpoints:

(43) a. $[\![\, \textit{between} \,]\!] = \lambda x \lambda y \lambda z \,.\, \exists p[\textbf{start}(p) = x \wedge \textbf{end}(p) = y \wedge z \in p]$

b. $[\![\, \textit{between Los Angeles and New York} \,]\!]$
$= \lambda z \,.\, \exists p[\textbf{start}(p) {=} \textbf{Los-Angeles} \wedge \textbf{end}(p) {=} \textbf{New-York} \wedge z {\in} p]$

Of course, given this denotation, non-path subjects are possible because z here is not a path but simply an individual located along one. To accommodate path subjects, it seems to be necessary to assume some polysemy. The denotation of the path variant would actually be simpler than (43a)—it simply requires introducing the path as an argument:

(44) $[\![\, \textit{between}_{\textit{path-variant}} \,]\!] = \lambda x \lambda y \lambda p[\textbf{start}(p) = x \wedge \textbf{end}(p) = y]$

In light of the similarity between the two, one might suspect that (43a) is derived from (44) by a type shift. The type shift required would, in fact, rather resemble one that is standardly assumed in the Vector Space Semantics strand of the locatives literature. Replacing vectors with paths, the vector space type shift applies to a predicate of paths and yields a predicate of individuals located at the end of those paths. What we would need here is more general, one that yields predicates of individuals located at *any point* along those paths:

(45) $[\![\, \textsc{vector-space-style shift} \,]\!] = \lambda P_{\langle p,\, t\rangle} \lambda x \,.\, \exists p[P(p) \wedge x \in p]$

(for the sake of argument)

But this would be a mistake. Going down that road would mean that arbitrary path predicates could be shifted this way, leaving us with no explanation for why non-path subjects like *Chicago* are impossible with path predicates like *from Los Angeles*. It truly seems to be a lexical semantic idiosyncrasy of *between* that it allows both sorts of subjects.

Another difference between *from/to* and *between/and* that's noted in some of the literature on modified numerals is that the former include their endpoints and the latter excludes them. We will return to this in section 4.7.

One further note about *between/and*. For strictly locative uses, the denotation we have provided won't suffice. That's because on such readings, it's possible for *between* to actually apply to only one nominal so long as it's plural:

(46) a. Chicago is between two cities.
b. The flight is between two Delta hubs.

That suggests that the denotation we provided, which *requires* two internal arguments, can't be right in general. But it is right for at least some range expressions, which don't always allow plural arguments:[7]

(47) a. Floyd saw between $\left\{\begin{array}{l}\text{ 5 and 10} \\ \text{*two odd numbers} \\ \text{*some odd integers}\end{array}\right\}$ capybaras.

b. The children's ages ranged between $\left\{\begin{array}{l}\text{ 5 and 10} \\ \text{??two fairly young ages} \\ \text{??some odd integers}\end{array}\right\}$.

c. This encyclopedia volume covers presidents between
$\left\{\begin{array}{l}\text{ Cleveland and Taft} \\ \text{??(two) portly gentlemen[8]}\end{array}\right\}$.

Indeed, to the extent that the ill-formed variant in (47c) can be accommodated, it requires construing the encyclopedia to cover presidents physically located between portly gentlemen.

For this reason, we will keep the denotation provided above as our official one for range-expression *between/and*. For the locative use, it suffices to modify *between* so that it applies to an arbitrary plural nominal and yields a property of individuals located along a path between the members of the

[7]It's worth noting that one could construe (47a) as evidence for another claim: that structures like (47a) are derived by ellipsis from a form like *Floyd saw between 5 ~~capybaras~~ and 10 capybaras*.

plurality:[9]

(48) $[\![\textit{between}]\!] = \lambda x : |x| = 2 \,.\, \lambda z \,.\, \exists p[\textbf{start}(p) \sqsubseteq x \wedge \textbf{end}(p) \sqsubseteq x \wedge z \in p]$

Requiring as a presupposition that the plurality have precisely two members, as in (48), rather than at least two, is probably too strong. There is a prescriptive prohibition on using *between* with more than two arguments.[10] No one would bother to stigmatize the use if it didn't exist, but we will leave this denotation as it is because pursuing the issue would take us too far afield.

3.3 *Through*

There is a third building block of range expressions that has a locative foundation: *through*. In its locative use, it often occurs as a part of a *from/to* structure as in (49):

(49) The snowplow cleared (from) 10 Main Street through (to) Elm Street.

There are a number of fine-grained idiosyncrasies in how *through* is used in locatives and elsewhere, some of which we will sidestep. The most pressing question to address is what *through* actually contributes, both in its independent use and when it occurs inside a *from/to* structure.

To appreciate the effect of *through*, it helps to consider the circumstances under which the *through*-less variant in (50) can be true:

(50) The snowplow cleared from 10 Main Street to Elm Street.

This is certainly true if there is a stretch of Main Street that starts at 10 Main Street and ends at the intersection with Elm Street, and that stretch has been entirely cleared. But that isn't actually necessary. Plowing snow is a complicated affair, and occasionally a parked car or other obstacle requires skipping over a portion of the street. If some such obstacle meant that a

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[9] We use $\sqsubseteq$ here to indicate the Link (1983)-style individual part relation that holds between a plurality and its members. The form in (48) slightly abuses our convention of using individual terms to refer to corresponding locations by assuming it works the other way around as well. Here's the fully explicit variant:

(i) $$[\![\textit{between}_{\textit{path-variant}}]\!] = \lambda x : |x| = 2 \,.\, \lambda z \,.\, \exists p \exists x' \exists x'' \begin{bmatrix} x' \sqsubseteq x \wedge x'' \sqsubseteq x \wedge \\ \textbf{start}(p) = \textbf{location}(x') \wedge \\ \textbf{end}(p) = \textbf{location}(x'') \wedge \\ z \in p \end{bmatrix}$$

[10] Prescriptivists recommend *among* instead for those cases, an injunction that if followed would lead to oddities like #*All flights among New York, Washington, and Delaware were canceled* and #*There's a town that's equidistant among New York, Washington, and Delaware.*

proper subpart of the path in (50) had to be skipped over—say, from 16 to 20 Main Street—(50) can nevertheless be judged true.

Caution is called for. It's not completely obvious that the judgment should be understood as being that the sentence is true, or simply that it is close enough to being true for the pragmatic circumstances. This notion—that sentences that are strictly false can be regarded as close enough to being true for current purposes—is at the heart of the Lasersohn (1999) theory of loose talk and imprecision, in which every expression in a sentence has a 'halo' around its denotation consisting of elements that are more or less interchangeable with it for current purposes. Thus *6 feet* denotes a precise measure, but when discussing someone's height, on might take a microscopic fraction of an inch below 6 feet to count as close enough. Only a pedant would object that they had been lied to when a person that was claimed to be six feet tall turns out to be a thousandth of an inch shorter. Perhaps that's what's happening in (50) on the relevant reading? Perhaps if the snowplow had to skip a few houses, that *does* make the sentence strictly false after all, but it might sometimes count as close enough to true?

This matters in part because it would instantly suggest an analytical course. Part of the Lasersohn vision is that halos can be more or less narrow, and certain expressions he calls slack regulators can constrict the size of the halo. With respect to time, *exactly* is one such expression: 3:01 might count as *3 o'clock* in some contexts, but in most it wouldn't count as *exactly 3 o'clock*. Likewise, *through* might signal that there that one should use a narrower halo around a *from/to* range, tolerating fewer exceptional gaps in the range.

One reason to suspect that this isn't the right course is that known slack regulators like *exactly* don't have this effect. *#?Exactly from 10 Main Street to Elm Street* definitely doesn't mean this, and it's not altogether clear that it means anything at all. If it's well-formed, it's only barely.

Another indication that *through* isn't a slack regulator is its position. It can occur just left of *to*, or in place of it, as (49) shows. But it can't occur on the extreme left, where we'd normally expect a slack regulator to occur in English:

(51) *The snowplow cleared through (from) 10 Main Street (to) Elm Street.

It is possible on the left only in one-sided ranges, where it can still occur to the left of *to*:

(52) The snowplow cleared through (to) Elm Street.

In the absence of *to*, the *through* is required for grammaticality, which may also be an indirect sign that it isn't fundamentally a slack-regulating modifier.

Those are syntactic considerations, but there is also a semantic argument. First, consider how slack regulation/imprecision works in bets. If Floyd bets Clyde that the next person to walk into the room will be six feet tall, Floyd loses the bet if that person is even a thousandth of an inch shorter. It may be a bit pedantic or unsporting to insist on this level of precision, but it's clearly not wrong. The terms of the bet are clearly not met if the person falls short of six feet by any amount. Next, let's change the terms of the bet to the content of (49): Floyd bet Clyde that the snow plow cleared from 10 Main Street to Elm Street. The way to lose the bet is for the snow plow to start past 10 Main Street or to stop short of Elm Street. But it's not at all clear that Floyd loses the bet if the snow plow missed a spot along the way or deliberately skipped one because it was impossible to plow. The bet seems to be simply underspecified with respect to exception tolerance. So *through* doesn't seem to regulate slack in the way that *exactly* does.

There is an alternative course. What *through* seems to do is indicate a relation between a path and another path provided by context that contains it. It requires that the path include a continuous stretch of the superpath, from one point to another. In the plow case, it makes sense to think of a master list of addresses along a street that are to be plowed. A path from 10 Main Street to 30 Main Street could be said to be *through* these addresses if it doesn't skip over any addresses on the master list. And conversely, if it does skip over any, it's a *from/to* path, but not a *through* path.

Of course, the master list can take various forms, and accordingly, what counts as a *through* path can change. If the master list is not of addresses but, say, of yard-long stretches of the street, a *through* path would need include every yard of the street from its start point to its end point.

Putting all this together, crucial part of the snowplow example in (49) would have the denotation in (53), where p' is the contextually provided implicit superpath argument of *through*:[11]

(53) $[\![$ *from 10 Main Street through*$_{p'}$ *to Elm Street* $]\!]$

$$= \lambda p \left[\begin{array}{l} \textbf{start}(p) = \textbf{10-Main-Street} \wedge \textbf{end}(p) = \textbf{Elm-Street} \wedge \\ p \subseteq p' \wedge \forall x[x \in p' \wedge \textbf{start}(p) \geq_{p'} x \geq_{p'} \textbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

The first component to highlight is the start and end. Those are provided by the *from/to* component. The *through* component provides everything else. It includes a conjunct that requires that the *through* path be a part of the

[11] Because there are in principle two orders involved here—the one for p and for p'—at issue, we indicate which path the order is associated with with a subscript, as in $\geq_{p'}$. Some additional assumptions about subpaths: we will write $p \subseteq p'$ to mean p is a subpath of p'; this requires that the elements of p be a subset of the elements of p' and that the ordering relation associated with p and p' agree on any element they both contain.

contextually-provided superpath p'. That bit might well be a presupposition, though we set that aside here for simplicity. The crucial conjunct is the next one. It requires the *through* path be a continuous stretch of the superpath—one that leaves nothing out, one that has no gaps. Everything in the superpath from the start of the *through* path to its end must be included.

Compositionally, this makes it possible to provide a denotation for *through*:

(54) $[\![\textit{through}_{p'}]\!]$
$$= \lambda x \lambda p \left[\begin{array}{l} \mathbf{end}(p) = x \wedge p \subseteq p' \wedge \\ \forall x [x \in p' \wedge \mathbf{start}(p) \geq_{p'} x \geq_{p'} \mathbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

This actually incorporates the meaning of locative *to* directly into *through*. One could instead treat *through* as an intersective modifier that contributes only the information about the contextually-supplied superpath, but its distribution seems to argue against that. *Through* can't combine with just any property of paths. If it could, we might expect it to modify a *from* PP (**through from Cleveland*) or a *between* PP (**through between Cincinnati and Cleveland*). We take the *to* that it can optionally occur with to be vacuous, its meaning now completely absorbed into *through*:

(55) $[\![\textit{through}_{p'} \textit{ (to) Elm Street}]\!]$
$$= \lambda p \left[\begin{array}{l} \mathbf{end}(p) = \mathbf{Elm\text{-}Street} \wedge \\ p \subseteq p' \wedge \forall x [x \in p' \wedge \mathbf{start}(p) \geq_{p'} x \geq_{p'} \mathbf{end}(p) \rightarrow x \in p] \end{array} \right]$$

This PP in turn can combine with a *for* PP in a variety of ways. In most of the locative examples we have so far considered, the full *from/to* structure is not itself a constituent but rather two adjacent PPs. In these cases, a property of paths VP meaning could combine intersectively with a *for* PP and then with (55). In principle, though, there is nothing preventing (55) from combining intersectively directly with a *for* PP, since both denote properties of paths.

3.4 *Summary so far*

This section has proposed a semantics for locative uses of *from*, *to*, *between*, and *through* framed in terms of paths. For some of these cases, there are alternatives in principle. Kracht (2002), for example, analyzes a class of prepositions that includes *between* in terms of regions rather than paths. The preceding discussion has, we hope, demonstrated that this conclusion is not unavoidable. And given our current goals, it's especially useful to avoid it, because it's more straightforward to generalize paths beyond the locative domain than it it is for regions. The range expressions that these prepositions build are fundamentally about navigating paths crosscategorially, across semantic types and syntactic categories.

This discussion leaves a number of important larger issues unaddressed. These include how path descriptions can serve as arguments and how they are interpreted when adjoined to VP, where it's not clear that the VP itself must denote a path description. We set both aside for the moment because our ultimately goal is a semantics for ranges, not for locative uses in particular. The question of how they serve as arguments will reemerge.

The next challenge, then, is to provide a semantics for non-locative uses range expressions, grappling with some of the special challenges they raise.

4 Paths beyond locations

4.1 *Generalizing paths*

Having laid the locative foundations of the semantics of ranges, it's now possible to provide a semantics for range expressions more broadly.

The first move will be to generalize points and paths. Just as a directed path can chart a course in a two- or three-dimensional space, it can chart a course in any domain that is (or can be construed to be) one-dimensional. That's where non-locative uses of range expressions are especially useful. The prototypical uses are numerical, where the path is, intuitively, on a number line. But of course paths can be across times as well, another one-dimensional domain. Likewise for degrees—although the whole domain of degrees isn't totally ordered, any particular degree participates in a scale that does have a linear order. There the generalization is straightforward. Things are perhaps slightly trickier in cases where individuals themselves are ordered, as in for example a line of people arranged alphabetically. But for the most part, a linear order is a linear order, and individuals linearly ordered aren't that different—for the purposes of defining ranges—than points in space, times, or degrees.

For the sake of explicitness, a few additional words about types. We'll assume that the model includes a domain of paths, D_p. In order to distinguish them from purely locative paths, we will call these RANGE PATHS. These are not an atomic type. A range path is simply a linearly ordered set composed of members of an atomic type. In the locative case, it's simply points or locations in space (members of D_l), but it could also be degrees (D_d), including numbers, which we take to be a special case of degrees. And it could be times (D_i) or individuals (D_e). In (56) this is stated more formally ($\mathscr{P}$ is the power set symbol):

(56) a set S with an ordering $\preceq$ is a member of D_p iff
$S \in \mathscr{P}(D_l) \cup \mathscr{P}(D_d) \cup \mathscr{P}(D_i) \cup \mathscr{P}(D_e)$ and $\preceq$ is a total order

This is in addition to standard definitions of atomic and functional types. As mentioned in 2.1, one membered ranges don't seem to be possible, and one could build that into (56), but it might be better to treat this effect as pragmatic rather than to build it into a type definition.

Such an approach is mostly a generalization of locative paths, but not exactly. Zwarts (2005) defines paths as functions from a real number from 0 to 1 inclusively to a point in space. We avoid this only because it treats paths in general as dense—that is, as having a point in the path between any two other points in it. That isn't the case for paths composed of individuals such as US presidents. It might be sufficient for our purposes to suppose that path functions can have any real number between 0 and 1 in their domain *in principle*, but some are partial functions. The range of the function would have to be extended to elements of any atomic type.[12]

The definition in (56) also bars mixed paths, ones that are composed of some combination of, for example, individuals and numbers. That seems sensible a priori, and we can find no linguistic evidence for such paths. One might imagine various far-fetched possibilities that superficially seem to require it—perhaps a list of index cards with concept descriptions on each of them—but examples like this might be better construed as involving the individual correlates of elements of other types.

It seems to be possible to use range expressions across a number of syntactic categories (Morzycki & Gobeski (2023)). That may suggest that it's too restrictive to build paths only out of members of an atomic type. A PP example is in (57a); an AP example, in (57b); and a degree word example in (57c):

(57) a. There was plankton at all levels of the ocean, from just under the surface to far below it.
b. The volunteers ran the gamut from slightly annoying to utterly intolerable.
c. Floyd was somewhere between barely and profoundly skeptical.

These examples seem to be strong evidence in favor of generalizing the

[12] This approach would also allow a single element to appear in a path more than once, at different locations in the path. That might be desirable for locative uses, but it seems less so for individual uses. See the discussion of Grover Cleveland at the close of this subsection. For the sake of explicitness, a definition along these alternative lines would have a function be a member of D_p iff its domain is the real numbers (or a proper subset thereof) and its range any of D_l, D_d, D_i, or D_e.

definition of paths, but we will not pursue this path here.

There is another respect in which this system may be too restrictive. It's not actually altogether clear that a linear (i.e., total) order is required. It's quite natural to define ranges among American presidents, for example. But American presidents are not totally ordered, or indeed in the strict sense ordered at all, because the ordering relation among them is not antisymmetric. That's because Grover Cleveland served nonconsecutive terms. He both preceded and followed Benjamin Harrison despite being, if historical sources are accurate, not identical to him. That said, this fact is an odd bit of trivia, little known and, like Cleveland, often forgotten. When range expressions are used in such contexts, it seems reasonable to suppose that the speaker actually has in mind a linear order, irrespective of what the facts may be. If one is asked to list all the presidents through Cleveland, it's not actually obvious whether to included Harrison or not. This doesn't seem to be a matter of ambiguity. It's not that the instruction is ambiguous between excluding and including Harrison. Rather, once the complication is pointed out, one has the sense of resignation normally associated with a sentence that one suddenly discovers to have a false presupposition.[13] A similar point could be made about alphabetized names, where confusion around prefixes like *de* or *von* can lead to errors in which a name both precedes and follows itself. We can refer to ranges across such a list, but only by thinking in terms of items in the list rather than names.[14]

4.2 *Predicates of ranges*

Section 2.1 observed that *from/to* range expressions are predicates of ranges, in the sense that they can't be predicated of a single range element:

(58) a. #7 is from 5 to 10.
b. #Chester A. Arthur is from Lincoln to Kennedy.

This seems to be a case of a fact already encountered for locatives in section 3.1:

(59) a. #Chicago is from Los Angeles to New York.
b. #Chicago is to New York.

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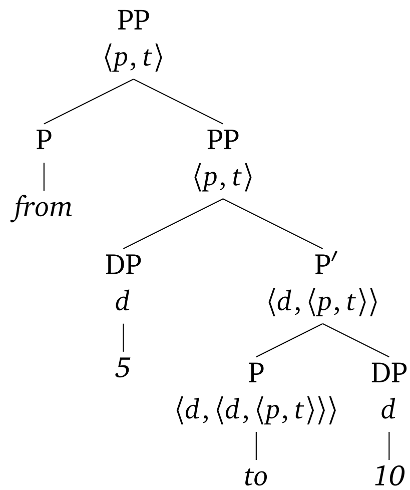
[13]There is even a convention for imposing a linear order on US presidents. They are conventionally numbered, and Cleveland is taken to be both 22nd and 24th.

[14]For locative uses, though, it might be necessary to assume paths in which the same point can occur repeatedly to account for round trips and the like.

All that needs to be done now is to spell out the denotation more explicitly, and to add one empirical wrinkle.

In locative uses, there are typically too distinct PPs, one headed by *from* and the other by *to*. With range expressions, there seems to be single *from/to* structure with a fixed internal order, as section 2.1 noted. Its syntax, we suggest, involves a head preposition, *from*, that takes a PP complement.[15] That PP complement requires a an overt specifier for type reasons, as in (60):

(60)



The specifier is necessary because otherwise the type of the matrix PP would be $\langle d, \langle p, t\rangle\rangle$, which would lead to a type clash in most PP positions.[16] We don't have strong evidence to determine how to distribute the semantics across the two prepositions, but we will built it into this version of *to* and leave *from* uninterpreted. (It can, after all, often be omitted.)

(61) $[\![\, to_{range} \,]\!] = \lambda d \lambda d' \lambda p[\textbf{start}(p) = d' \wedge \textbf{end}(p) = d]$

The impossibility of subjects that fail to denote range paths—such as *7* in (58)—follows from the fact that the PP is defined only for path subjects.

In locatives uses, potential subjects were expressions like *the trip*, which could be construed as inherently denoting paths. Something similar is possible in non-locative cases with nouns like *the range* or *the interval*:

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[15]Prepositions with PP complements are not unusual in English: *from under the bed*, *out of the window*.

[16]One could engineer this even more effectively by supposing that *from* actually denotes an identity function of type $\langle\langle p, t\rangle, \langle p, t\rangle\rangle$, thereby creating a type clash irrespective of the position of the PP.

(62) The $\begin{Bmatrix}\text{range}\\\text{interval}\end{Bmatrix}$ was from 5 to 10.

But these simple cases are just the starting point. There are also cases in which it's less clear that the subject directly denotes a path:

(63) a. The children's ages were from 5 to 10.
b. The estimated weights were from 5 to 10.

It's certainly possible in principle to construe *the children's ages* as a path: it could simply be a set of degrees of oldness, such as {2 years old, 4 years old, 5 years old}, and one can treat *the estimated weights* likewise. But such representations do seem to lose the underlying complexity of degree nominalizations. There are good reasons to doubt that degree nominalizations refer directly to a degree (see Moltmann 2009 and references there) even in the singular case. In (63b), for example, it seems odd to suppose that a particular degree of weight—say, 150 pounds—has the property of being estimated. In both examples in (63), an additional component of the meaning has to come from plural morphology. So one way or another, the picture of how these nominalizations come to refer to paths has to be more complicated.

Indeed, even the example we offhanded treated as a path-denoting nominal—*the trip*—is not obviously only path-referring. One can enjoy a trip or regret a trip, but one can't enjoy simply a path in space or regret a path. Both these points demonstrate that something more must be said about how nominals come to denote paths. We will conceptualize these operations as the result of type shifts. In the case of expressions like *age*, much hinges on what one assumes to be the underlying semantics of the degree nominalization itself. Addressing that would take us far afield. But for the sake of explicitness, it's worth sketching how things might work. The starting point is to suppose, for the sake of simplicity alone, that *age* is a predicate of degrees. The plural *ages* might denote a plurality of degrees, a notion for which there is independent evidence from other domains (Dotlačil & Nouwen 2016). The full nominal *the children's ages* would thus refer to the maximal plurality of degrees that consists of (contextually relevant) children's ages. The mapping to a path, then, is simple: it's that plurality of degrees represented as a set, ordered by the inherent ordering provided by the scale of which these degrees are members.

In the case of eventive nouns like *trip*, the crucial element of the shift to path reference is provided by an appropriate trace function, in the sense of Link (1998): either the temporal trace function τ that maps an event to its running time, or its locative counterpart.

That's a lot of type shifting, and as it turns out there is more in store. In both of these cases, though, the shift is a simple and natural one. And in both cases, it is one whose precise character involves far more than the grammar of paths: in one case, the thorny puzzle of what degree nominalizations denote, and in the other the grammar of events. These shifts simply allow us to evade this distinct issues. There is, of course, an alternative, as there often is with type shits: instead of the shift, one might use a functional head. We don't have evidence against such a strategy.

There is some reason, though, to avoid another conceivable analytical path: building the type shifts directly into the denotations of the range expressions. One might, for example, take *from Cincinnati to Cleveland* to denote not a property of paths that start at Cincinnati and end at Cleveland, but rather a property of *events* that do so. The trouble is that this strategy would require building into the range expression at least two different type shifts: the eventive one, and the degree one considered above. The result would be that every range expression would be polysemous between being a predicate of events and of degrees. Indeed, times are also a possibility. Moreover, the different flavors of range expression (*from/to*, *between/and*, *through*) would all need to be polysemous in exactly the same way. That's not inconceivable, of course, but it is also not the most parsimonious course.

4.3 *Cardinality*

As we have seen, many uses of range expressions are about cardinality:

(64) Floyd saw five to ten capybaras.

It's now possible to make sense of this phenomenon in terms of the analysis so far.

First, some assumptions about cardinality ascriptions. Following a long tradition, we take cardinal numerals as measure phrases arguments of an implicit adjective (Bresnan 1972, Hackl 2000; see Morzycki 2016 for further references and discussion). This is standardly taken to be a form of *many*, so that *five capybaras* is underlyingly *five* MANY*capybaras*. The implicit adjective denotes a relation between a degree provided by the numeral and a plural individual ($|\cdot|$ maps an individual to its cardinality):

(65) a. $[\![\text{MANY}]\!] = \lambda d \lambda x [|x| = d]$

b. $[\![\textit{five}]\!] = \mathbf{5}$

c. $[\![\textit{five}\ \text{MANY}\textit{capybaras}]\!] = \lambda x [|x| = \mathbf{5} \wedge \mathbf{capybaras}(x)]$

Thus, as in (65b), *five* MANY denotes a property of individuals that can be interpreted intersectively with *capybaras*. This can be existentially closed by an implicit indefinite quantifier.

It will be necessary to deviate from this standard picture in one important way. In an analysis of modified numerals, Nouwen (2010) adopts a variant of MANYthat builds in an additional ingredient: an 'exactly' semantics. We will follow suit. The standard version has an 'at least' semantics. If in fact Floyd saw five capybaras, it's true that there's a capybara plurality that he saw with a cardinality of five. But then it's also necessarily true that there is a capybara plurality that he saw with a cardinality of four—indeed, several such pluralities, depending on which fifth capybara is excluded. That means that whenever it's true that he saw 5, it's also true that he saw 4, and 3, and so on.

Nouwen's exactly variant, which we will call MANY_{MAX}, avoids this outcome. $\text{MANY}_{\text{MAX}}$is a comparative determiner, one that bundles adjective MANYand the implicit indefinite determiner into a single package, denoting a function from degrees to generalized quantifier determiner meanings:

(66) $[\![\text{MANY}_{\text{MAX}}]\!] = \lambda d \lambda P_{\langle e,t \rangle} \lambda Q_{\langle e,t \rangle} \,.\, \exists ! x[|x| = d \wedge P(x) \wedge Q(x)]$

The existential quantifier that would otherwise be provided by the implicit determiner is built in. Crucially, though, it is not an ordinary existential, but rather one that requires that there be exactly one individual that satisfies its nuclear scope. This gives rise to the 'exactly' semantics. In our capybara scenario, 5 is a possible value for d because there is exactly one five-membered plurality of seen capybaras. But 4 and other lower values are not possible, because if Floyd in fact saw five capybaras, there are multiple four-membered pluralities of seen capybaras.

At this stage, it makes sense to simply adopt MANY_{MAX}as a stipulation, and return later to why this choice was necessary.

For the sake of explicitness, (67) illustrates how the pieces come together in a simple cardinal numeral case:

(67) [[Five MANY_{MAX}] capybaras] ate Floyd.

a. $[\![\textit{[five } \text{MANY}_{\text{MAX}}\textit{] capybaras}]\!] = [\![\text{MANY}_{\text{MAX}}]\!]([\![\textit{five}]\!])([\![\textit{capybaras}]\!])$
$= \lambda Q_{\langle e,t \rangle} \exists ! x[|x| = \mathbf{5} \wedge \mathbf{capybaras}(x) \wedge Q(x)]$

b. $[\![\textit{[[five } \text{MANY}_{\text{MAX}}\textit{] capybaras]}]\!]([\![\textit{ate Floyd}]\!])$
$= \exists ! x[|x| = \mathbf{5} \wedge \mathbf{capybaras}(x) \wedge \mathbf{ate}(x, \mathbf{Floyd})]$

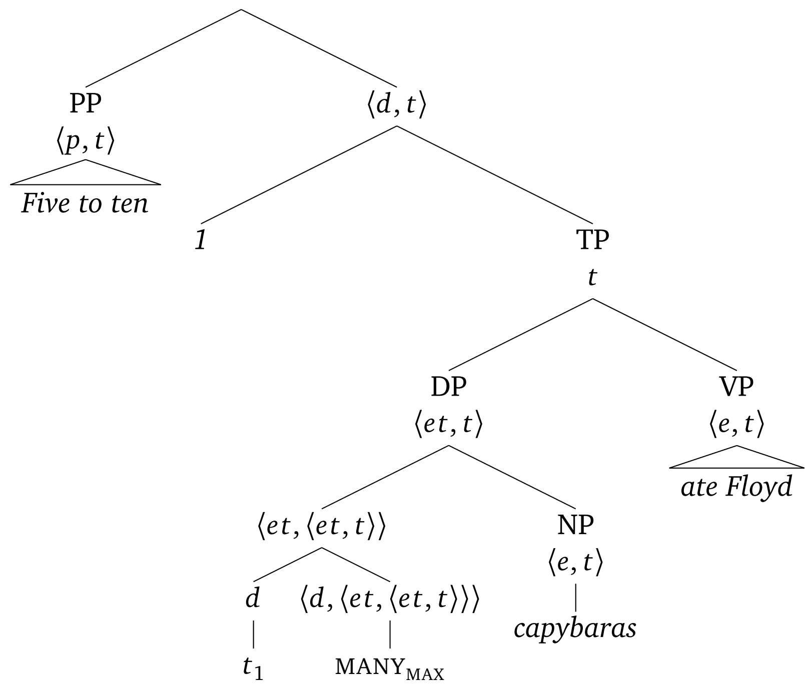
We have changed our pet example to embed the numeral inside the subject, thereby making it possible to avoid spelling out the complicating effect of quantifier raising. That's useful because, in a moment, there will be a need for an independent case of quantifier raising that would otherwise complicate the picture needlessly.

With this in place, it's now possible to ask what happens with a range expression in place of the numeral:

(68) a. Five to ten capybaras ate Floyd.
b. **LF:** [[[Five to ten] MANY_{MAX}] capybaras] ate Floyd

The short answer is a type clash. Our standing assumptions treat *(from) five to ten* as a property of paths. That clashes with MANY_{MAX}, which expects a degree. A standard response to a type clash is QR, and we propose to use it here too. In order to dodge the type clash, the displaced range expression needs to leave behind a trace that denotes a degree. That gives rise to the logical form in (69) (we represent the lambda abstraction triggered by movement in the Heim & Kratzer 1998 style as a numeric index directly in the tree):

(69)



The movement of the range expression triggers lambda abstraction in the usual way (Heim & Kratzer 1998), and in this case, because the trace is degree denoting the lambda must bind a degree. This leaves us in an interesting position. The lambda expression now denotes a property of degrees:

(70) $[\![$ *1 [[*t_1 MANY_{MAX}*] capybaras] ate Floyd* $]\!]$
$= \lambda d\,.\,\exists! x[|x| = d \wedge \textbf{capybaras}(x) \wedge \textbf{ate}(x, \textbf{Floyd})]$

More precisely, it's the property of being the exact number of capybaras that ate Floyd. This is a singleton property, because of the uniqueness existential.

This remedies the type clash at the bottom of the tree, but there remain two elements at the top of the tree that still can't combine straightforwardly.

One is the range expression, a property of paths. The other is the singleton property of degrees. To reconcile these pieces, it will take a few additional steps, at least one of which is similar to the type shift we already entertained in section 4.2. The truth conditions of the sentence require that the singleton degree to be included in the range—in path terms, to be no smaller than the start of the path and no greater than its end. One way to express this is to construe the degree property as a singleton set of degrees, and to say that this set must be a subset of the path. The natural way to implement this is with a type shift:

(71) $[\![\text{ PATH-SHIFT }]\!] = \lambda D_{\langle d,t\rangle} \lambda p \,.\, D \subseteq p$

In this case, it combines with the singleton degree property as in (72), but to accommodate the set-talk in (71), it makes sense to now represent this property as a set:

(72) $[\![\text{ PATH-SHIFT }]\!]([\![\,\textit{1 [[}t_1 \text{ MANY}_{\text{MAX}}\textit{] capybaras] ate Floyd}\,]\!])$

$$= \lambda p_p \,.\, \left\{ d \;\middle|\; \exists! x \begin{bmatrix} |x| = d \wedge \\ \textbf{capybaras}(x) \wedge \\ \textbf{ate}(x, \textbf{Floyd}) \end{bmatrix} \right\} \subseteq p$$

This yields a property of paths that contain the crucial singleton degree. That in turn means that this can combine intersectively with the property of paths expressed by the range expression:

(73) a. $[\![\,\textit{five to ten}\,]\!] = \lambda p_p \,.\, \textbf{start}(p) = \mathbf{5} \wedge \textbf{end}(p) = \mathbf{10}$

b. $[\![\,\textit{[five to ten]}\text{ PATH-SHIFT }\textit{1 [[}t_1 \text{ MANY}_{\text{MAX}}\textit{] capybaras] ate Floyd}\,]\!]$

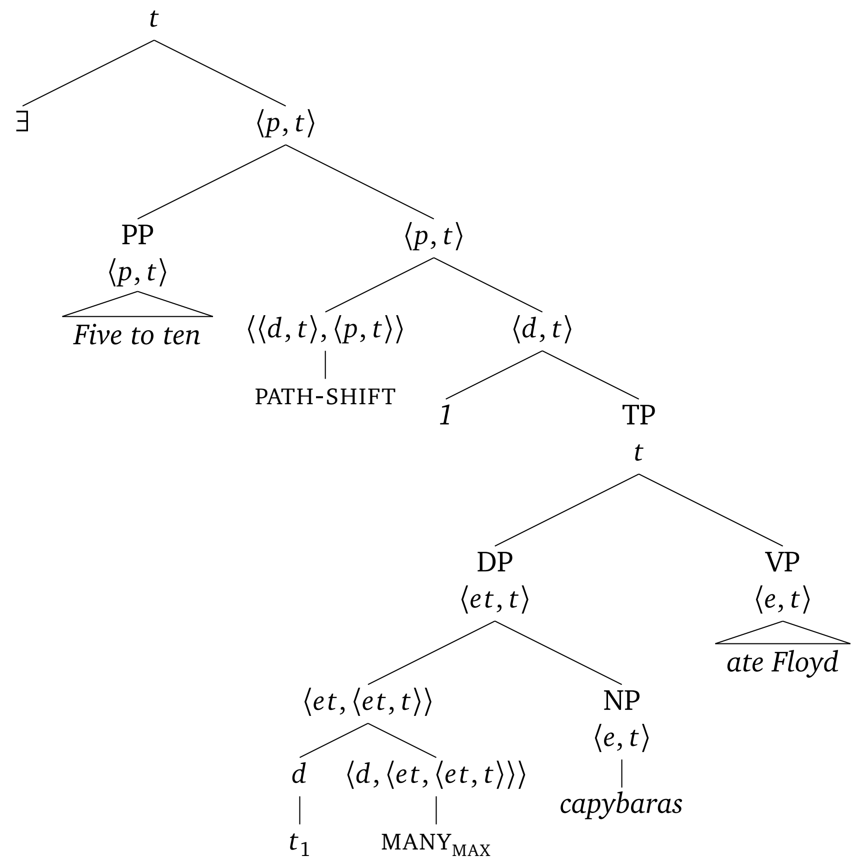
$$= \lambda p_p \,.\, \textbf{start}(p) = \mathbf{5} \wedge \textbf{end}(p) = \mathbf{10} \wedge \left\{ d \;\middle|\; \exists! x \begin{bmatrix} |x| = d \wedge \\ \textbf{capybaras}(x) \wedge \\ \textbf{ate}(x, \textbf{Floyd}) \end{bmatrix} \right\} \subseteq p$$

At this stage, a truth value is required because we have reached the top of the tree, but what we have in hand is a property of paths. The natural move, and one widely independently motivated from other domains, is existential closure. The sentence is true iff there exists a path that satisfies (73b). The closure operation is spelled out in its path-based form in (74) for the sake of explicitness, though one could also view this as a special case of a more general cross-categorial existential closure operation:

(74) $[\![\,\exists\,]\!] = \lambda P_{\langle p,t\rangle} \,.\, \exists p[P(p)]$

The tree in (75) is updated to include the two type shifts, and the full sentence denotation in (76):

(75)



(76) $[\![$ ∃ *[five to ten]* PATH-SHIFT *1 [[*t_1 MANY$_{\text{MAX}}$*] capybaras] ate Floyd* $]\!]$

$$= \exists p \left[\begin{array}{l} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \left\{ d \;\middle|\; \exists! x \left[\begin{array}{l} |x| = d \wedge \\ \mathbf{capybaras}(x) \wedge \\ \mathbf{ate}(x, \mathbf{Floyd}) \end{array} \right] \right\} \subseteq p \end{array} \right]$$

This yields the correct truth conditions: the sentence is true if there's a path that starts at 5 and ends at 10 and contains the exact number of capybaras that ate Floyd. This, of course, could be the case only if the exact number of capybaras was no smaller than 5 and no greater than 10.

We implemented this analysis with a simple type shift and a separate operation of existential closure. That isn't strictly necessary. We could have combined the two into a single more complicated type shift:

(77) $[\![$ PATH-SHIFT $]\!] = \lambda D_{\langle d,t \rangle} \lambda P_{\langle p,t \rangle} \, . \, \exists p[P(p) \wedge D \subseteq p]$

(alternative not adopted)

The advantage of separating the two is that existential closure is an operation for which there is a great deal of independent evidence in the grammar, and that the resulting type shift truly seems like a type shift, with only a minimal semantic contribution of its own. That type shift is not itself widely independently motivated, but it is also so minimal a move that it makes sense to think of it as an aspect of the broader grammatical logic of paths.

To close this section, a few words about MANY_{MAX}. The analysis provided above would not have been possible if we had used ordinary MANY, with its 'at least' semantics. Had we used it, the set of degrees expressed by *1 [[*t_1 MANY_{MAX}*] capybaras] ate Floyd* would not be a singleton set. Instead, it would contain not only the maximal number of capybaras, but also every integer down to 1. That in turn would mean that that any path that contains this set would have to have to start at 1. But in order to reflect the effect of the range expression, it's crucial that the path start instead at the start of the range.

4.4 *Cardinalities, quantification, and scope*

An additional and rather hairy compositional challenge awaits. The analysis in the previous section articulated how cardinality and range expressions interact in simple sentences. Along the way, it was necessary to suppose that range expressions QR. But what does that predict about other quantifiers?

In particular, there needs to be an explanation of how, in quantified contexts, what we earlier called set punctual readings arise. As a brief reminder, the crucial observation is that in the scope of a quantifier as in (78), readings of range expressions arise in which there are multiple crucial values in the range:

(78) Every tourist saw five to ten capybaras.

In this case, each tourist may have seen a different number of capybaras, so long as it's the case for every tourist that their count of capybaras falls in the range.

This will turn out to be a matter of quantifier scope. The computation is has a number of (literally) moving parts. First, (78) requires the range expression to occur in object position, so the indefinite *five to ten capybaras* will need to scope out on its own. Second, there is the crucial matter of how *every tourist* scopes. Third, there is the independent movement of the range expression.

Leaving the subject in situ for the moment, (78) would have the LF in (79):

(79) ∃ [five to ten]
[PATH-SHIFT [1 [t_1 MANY$_{\text{MAX}}$ capybaras] [2 [every tourist saw t_2]]]]

This scopes out the indefinite, and then scopes the range expression outside of it. The semantics is as in (80):

(80) a. $[\![$ *2 every tourist saw* t_2 $]\!] = \lambda x \,.\, \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y, x)]$

b. $[\![$ t_1 MANY$_{\text{MAX}}$*capybaras* $]\!]$

$$= \lambda Q_{\langle e,t\rangle} \,.\, \exists! x[|x| = [\![t_1]\!] \wedge \mathbf{capybaras}(x) \wedge Q(x)]$$

c. $[\![$ t_1 MANY$_{\text{MAX}}$*capybaras* $]\!]([\![$ *2 every tourist saw* t_2 $]\!])$

$$= \exists! x \begin{bmatrix} |x| = [\![t_1]\!] \wedge \mathbf{capybaras}(x) \wedge \\ \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y, x)] \end{bmatrix}$$

d. $[\![$ *1 [*t_1 MANY$_{\text{MAX}}$*capybaras] [2 [every tourist saw* t_2*]]* $]\!]$

$$= \lambda d \,.\, \exists! x \begin{bmatrix} |x| = [\![t_1]\!] \wedge \mathbf{capybaras}(x) \wedge \\ \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y, x)] \end{bmatrix}$$

e. $[\![$ PATH-SHIFT $]\!]([\![$ *1 [*t_1 MANY$_{\text{MAX}}$*capybaras] [2 [every tourist saw* t_2*]]* $]\!])$

$$= \lambda p_p \,.\, \left\{ d \;\middle|\; \exists! x \begin{bmatrix} |x| = d \wedge \\ \mathbf{capybaras}(x) \wedge \\ \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y, x)] \end{bmatrix} \right\} \subseteq p$$

f. $[\![$ *five to ten* $]\!] = \lambda p_p \,.\, \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10}$

g. $[\![$ *∃ [five to ten] [*PATH-SHIFT *[1 [*t_1 MANY$_{\text{MAX}}$ *capybaras] [2 [every tourist saw* t_2*]]]]* $]\!]$

$$= \exists p \begin{bmatrix} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \left\{ d \;\middle|\; \exists! x \begin{bmatrix} |x| = d \wedge \\ \mathbf{capybaras}(x) \wedge \\ \forall y[\mathbf{tourist}(y) \rightarrow \mathbf{saw}(y, x)] \end{bmatrix} \right\} \subseteq p \end{bmatrix}$$

This yields a reading under which there's a particular cardinality of capybaras that all the tourists saw, and it's contained in a path from 5 to 10. This amounts to saying that the tourist that saw the fewest capybaras saw a number in the range. That's a possible reading, but it's not the most interesting or natural one—nor is it a set punctual reading.

The set punctual reading requires scoping out the universal quantifier too, to give it wide scope over the existential quantifier contributed by MANY$_{\text{MAX}}$, as in the LF in (81):

(81) [every tourist] 3 [∃ [five to ten]
[PATH-SHIFT [1 [t_1 MANY$_{\text{MAX}}$ capybaras] [2 [t_3 saw t_2]]]]]

The computation is in (82), abbreviated for the sake of sanity:

(82) a. $\left[\!\left[\begin{array}{l} \textit{3 } \exists \textit{ [five to ten] [}\textsc{path-shift}\textit{ [1 [}t_1 \text{ MANY}_{\text{MAX}} \textit{ capybaras]} \\ \textit{[2 [}t_3 \textit{ saw } t_2\textit{]]]]} \end{array} \right]\!\right]$

$$= \lambda y \,.\, \exists p \left[\begin{array}{l} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \left\{ d \;\middle|\; \exists! x \left[\begin{array}{l} |x| = d \wedge \\ \mathbf{capybaras}(x) \wedge \\ \mathbf{saw}(y, x) \end{array} \right] \right\} \subseteq p \end{array} \right]$$

b. $[\![\textit{every tourist}]\!] = \lambda P_{\langle e,t \rangle} \lambda Q_{\langle e,t \rangle} \,.\, \forall y [P(y) \rightarrow Q(y)]$

c. $\left[\!\left[\begin{array}{l} \textit{every tourist [3 } \exists \textit{ [five to ten] [}\textsc{path-shift}\textit{ [1 [}t_1 \text{ MANY}_{\text{MAX}} \textit{ capybaras]} \\ \textit{[2 [}t_3 \textit{ saw } t_2\textit{]]]]]} \end{array} \right]\!\right]$

$$= \forall y \left[\mathbf{tourist}(y) \rightarrow \exists p \left[\begin{array}{l} \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \\ \left\{ d \;\middle|\; \exists! x \left[\begin{array}{l} |x| = d \wedge \\ \mathbf{capybaras}(x) \wedge \\ \mathbf{saw}(y, x) \end{array} \right] \right\} \subseteq p \end{array} \right] \right]$$

The result a reading in which for every tourist, there is a potentially different exact number of capybaras seen, and each such number is contained in the range from 5 to 10.

This provides a theory of set punctual readings in general. These arise as simply the consequence of wide-scope universal quantifiers. That seems a simple explanation, and it's consistent with much of the data. It does have the interesting side-effect, though, that something must be said about how these readings arise with plural subjects in the absence of an overt quantifier. A natural answer is available, though: a implicit quantifier, such as a distributivity operator.

4.5 *Singleton punctual reading with overt adjectives and maximality*

We now have an analysis of both singleton and set punctual readings of cardinality expressions. But what about other forms of measure phrase? The answer turns out to have consequences for theories of the lexical semantics of adjectives.

A basic example of a range expression serving as a measure phrase to an overt adjective is in (83a), which has the LF in (83b):

(83) a. Floyd is from five feet to ten feet tall.

b. **LF:** ∃ [from five feet to ten feet] PATH-SHIFT 1 Floyd is [t_1 tall]

It's worth acknowledging from the start that it's more natural to avoid repetition of *feet*, preferring (84) over (83):

(84) Floyd is from five to ten feet tall.

Providing a semantics for (84), though, requires making assumptions about the internal grammar of measure phrases without shedding much additional light on range expressions, the topic at hand.

The most common analysis of gradable adjectives treats them as relations between individuals and degrees, as in (85a), yielding sentence denotations like (85b):

(85) a. $[\![\, \textit{tall} \,]\!] = \lambda d \lambda x \,.\, \mathbf{tall}(x, d)$

b. $[\![\, \textit{Floyd is six feet tall} \,]\!] = \mathbf{tall}(\mathbf{Floyd})(\mathbf{6ft})$

The measure phrase position can be occupied by a range expression. Because the adjective expects to find a degree in the measure phrase position, range expressions can't be directly interpreted there because they denote properties of paths. Here, the story largely recapitulates what we have already encountered for cardinality cases. The range expression must QR to a position where it can be interpreted with the aid of PATH-SHIFT as in the LF in (86):

(86) **LF:** ∃ [from five feet to ten feet] PATH-SHIFT 1 Floyd is t_1 tall

This means that the clause from which the range expression emerged will denote a property of degrees, as in (87a), and the matrix clause will have the denotation in (87b):

(87) a. $[\![\, \textit{1 Floyd is } t_1 \textit{ tall} \,]\!] = \lambda d \,.\, \mathbf{tall}(\mathbf{Floyd}, d)$

b. $[\![\, \exists \textit{ [from five feet to ten feet]} \text{ PATH-SHIFT } \textit{1 Floyd is } t_1 \textit{ tall} \,]\!]$

$$= \exists p \begin{bmatrix} \mathbf{start}(p) = \mathbf{5ft} \wedge \mathbf{end}(p) = \mathbf{10ft} \wedge \\ \{ d \mid \mathbf{tall}(\mathbf{Floyd}, d) \} \subseteq p \end{bmatrix}$$

The resulting denotation requires that the degrees to which Floyd is tall be part of a path from five feet to ten feet. That gets the semantics half right. It ensures that Floyd's height must be below ten feet, because if it were to exceed ten feet his heights wouldn't be part of such a path. But it provides the wrong result about his minimum height. The sentence should require that his height be above five feet. That's not what (87) says. That's because, on the most common assumptions, anyone that is six feet tall is also five feet tall, four feet tall, and so on. That in turn means that the set of degrees to which Floyd is tall in (87) includes many degrees going all the way down to zero. So the semantics in (87) requires that a set of degrees that goes all the way down to zero be part of a path that starts only at five feet. That can't be the case, so the denotation in (87) predicts the sentence would necessarily be false. It actually predicts that there is no way to constrain a height using the bottom end of a range expression.

The solution may be apparent, in light of the discussion of cardinality in section 4.3: there is a missing component of maximality. The set of degrees in (87) should be not one that goes from zero to Floyd's maximal height, but rather it should include *only* the maximal height. That accords with the major alternative conception of adjective semantics, in which maximality is built into the grammar of the adjective. That approach is found explicitly in Kennedy (1997) and Heim (2000) among many others for adjectives in general, and it is sometimes proposed specifically for adjectives with measure phrases in particular (notably in Schwarzschild 2005). As it turns out, evidence from range expressions points in this direction.

To demonstrate this, it will be helpful to change our representation of adjective meanings. Rather than treating the predicate **tall** as a relation between an individual and any degree to which they are tall, (88a) treats it as a a function from an individual to the *maximal* degree to which they are tall:[17]

(88) a. $[\![\textit{tall}]\!] = \lambda d \lambda x[\textbf{tall}(x) = d]$
b. $[\![\textit{Floyd is six feet tall}]\!] = [\textbf{tall}(\textbf{Floyd}) = \textbf{6ft}]$

Returning to the computation of the range expression sentence in (84), the result of this change propagates as in (89):

(89) a. $[\![\textit{1 Floyd is } t_1 \textit{ tall}]\!] = \lambda d[\textbf{tall}(\textbf{Floyd}) = d]$
b. $[\![\exists$ *[from five feet to ten feet]* PATH-SHIFT *1 Floyd is* t_1 *tall* $]\!]$

$$= \exists p\begin{bmatrix} \textbf{start}(p) = \textbf{5ft} \wedge \textbf{end}(p) = \textbf{10ft} \wedge \\ \{ d \mid \textbf{tall}(\textbf{Floyd}) = d \} \subseteq p \end{bmatrix}$$

This resolves the difficulty concerning lower bounds. The degree set is now a singleton set, containing only the maximal height of Floyd. That means that if Floyd is more than five feet tall, the singleton degree set will be part of a path that starts at five, rendering the sentence true, as it should be. If his maximal height is below five feet, there will be no appropriate path that the singleton degree set is part of, and the sentence will correctly come out as false.

The upshot, then, is that range expressions seem to provide evidence for the view that adjectival measure phrase interpretation encodes maximality. It's worth noting, though, that there is an alternative strategy available that

[17]For consistency, this leaves the type of adjectives unchanged, and it avoids introducing an explicit maximality operator into the semantics. Neither of these is crucial.

would allow us to avoid this result. It would build the crucial maximality component into PATH-SHIFT itself, as in (90):

(90) $[\![\text{PATH-SHIFT}]\!] = \lambda D_{\langle d,t \rangle} \lambda p \,.\, \mathbf{max}(D) \in p$ *(for consideration only)*

Thus PATH-SHIFT would require that the maximal degree in a shifted degree set be a member of the path. In some respects this might be a more parsimonious course, because it wouldn't push us toward particular assumptions about adjectives. Moreover, (90) is a priori as a type shift. The contribution it makes remains fairly minimal, reconciling some type trouble in the computation and doing so by introducing maximality. It's an appealing approach.

That said, it seems more appealing still to avoid positing maximality in the type shift if we can help it, as indeed we can. The maximality we need has been independently proposed for other reasons in both of the domains in which we need it: cardinality indefinites and adjectives with measure phrases. Given the choice, it seems more interesting to count range expressions as providing additional evidence for these established ideas than to go out of our way to avoid making interesting predictions. But of course it's ultimately a matter of taste.

4.6 *Interval ranges and exceptives*

We have now provided an analysis of singleton punctual readings and set punctual readings, with the difference being that set punctual readings arise automatically in the presence of quantifiers with wide scope. Next on the agenda are interval readings.

As a reminder, interval readings involve cases in which all of a range seems to be crucial. They are normally expressed in English with *through*, and unlike punctual readings they are compatible with exceptives:

(91) This volume covers U.S. presidents Kennedy through to Carter (except Nixon).

We have in hand a denotation for *through* in locative cases from section 3.3, so all that needs to be done is to ask how it extends to ranges more generally.

The idea for locative *through* is that it makes crucial reference to a contextually-provided superpath, represented as p' in the denotation, repeated here from section 3.3:

(92) $[\![\textit{through}_{p'}]\!]$

$$= \lambda x \lambda p \begin{bmatrix} \mathbf{end}(p) = x \wedge p \subseteq p' \wedge \\ \forall x [x \in p' \wedge \mathbf{start}(p) \geq x \geq \mathbf{end}(p) \rightarrow x \in p] \end{bmatrix}$$

This denotation requires that every element of the superpath that's between the endpoints of the its subpath be included in the subpath. No eligible elements of the superpath can be skipped. That's what makes it possible for a plow to clear snow from 10 Main Street through 20 Main Street as long as it skip over no addresses in the list of addresses along Main Street. The list, in this example, provides the superpath. Importantly, if the list is the superpath, it would allow the plow to skip any addresses that are not in the list itself, perhaps because there is no house with a particular number.

To extend this beyond locative uses, we need to loosen an assumption. The variable x in (92) must be able to range over any types of which a path may be composed, whether individuals, degrees, or something else. For the most part, that suffices to achieve what we need. In the American president case, for example, this ensures that presidents *Kennedy through to Carter* be a range that's part of the larger ordering of American presidents, and that it skip over none.

What remains to be explained is how the exceptive diagnostic works. Why is it that exceptives are possible with interval readings but not with punctual readings? The answer comes from a basic fact about the distribution of exceptives. As von Fintel (1993) shows, exceptives are systematically possible in the scope of a universal quantifier. Because the exceptives at issue target elements of ranges, this needs to be understood to mean universal quantification over range elements. Von Fintel (1993) encodes this exceptive denotations by making them sensitive to contextually-provided quantificational domain restrictions. For convenience, though, we will leave the universal quantifier restriction as a stipulation, yielding a fairly straightforward denotation (again, taking x to range over arbitrary path elements):

(93) a. $[\![\, \textit{except}_{p'} \,]\!] = \lambda x \lambda p : x \in p' \,.\, x \notin p$

b. $[\![\, \textit{except}_{p'}\ \textit{Nixon} \,]\!] = \lambda p : \mathbf{Nixon} \in p' \,.\, \mathbf{Nixon} \notin p$

As framed in (93a), *except* also imposes the presupposition that the excepted element be in the contextually-provided superpath. (This reflects a similarity between the role of the contextually-provided superpath and von Fintel's contextual domain restrictions.) This can combine intersectively with a range expression it modifies, as in (94):

(94) a. $[\![\, \textit{Kennedy through}_{p'}\ \textit{to Carter} \,]\!]$

$$= \lambda p \,.\left[\begin{array}{l} \mathbf{start}(p) = \mathbf{Kennedy} \wedge \mathbf{end}(p) = \mathbf{Carter} \wedge p \subseteq p' \wedge \\ \forall x[x \in p' \wedge \mathbf{start}(p) \geq x \geq \mathbf{end}(p) \rightarrow x \in p] \end{array}\right]$$

b. $[\![$ *Kennedy through*$_{p'}$ *to Carter except*$_{p'}$ *Nixon* $]\!]$

$$= \lambda p \ . \left[\begin{array}{l} \textbf{start}(p) = \textbf{Kennedy} \wedge \textbf{end}(p) = \textbf{Carter} \wedge p \subseteq p' \wedge \\ \forall x[x \in p' \wedge \textbf{start}(p) \geq x \geq \textbf{end}(p) \rightarrow x \in p] \wedge \textbf{Nixon} \notin p \end{array} \right]$$

The result is a property of paths starting at Kennedy, ending at Carter, skipping Nixon, all part of a larger superpath, here most naturally understood to consist of American presidents.

4.7 *Inclusive and exclusive ranges*

It is sometimes suggested that one difference between *from/to* and *between/and* is that *between/and* excludes its endpoints but *from/to* includes them. That is, *between 10 and 20* involves a range that includes values above 10 and under 20, and *from 10 to 20* involves a range that includes 10 and 20 themselves. That hasn't played a crucial role for us here, but some words on the topic are in order.

The first step will be to include some additional evidence. Whatever the default orientation of range expressions, it's possible to change it with adverbials:

(95) Floyd listed all the integers $\left\{ \begin{array}{l} \text{from 10 to 20 exclusive(ly)} \\ \text{between 10 and 20 inclusive(ly)} \end{array} \right\}$.

It's also worth pointing out that this really does seem to be a matter of defaults. Psycholinguistic experiments often invite sentences such as (96):

(96) Participants answered on a Likert scale between 1 and 5.

The intended reading is normally that 1 and 5 are possible ratings. Likewise, if Oone describes an encyclopedia volume as in (97), one would be surprised to find that 1700 is contained in the previous volume:

(97) This volume covers between 1700 and 1800.

An inclusive reading is also natural in (98):

(98) We invited between 9 and 10 people.

If (98) were strictly exclusive, it would require inviting fractional guests. And of course, if I ask you to keep some confidential information *just between you and me*, I don't intend that neither of us should know about it.

That all suggests that rather than hard-wiring the distinction into *between/and* and *from/to*, it should be left flexible and underspecified. It seems likely that *between* normally defaults to exclusive readings, but that may well be a statistical tendency have to do with frequency of use rather than lexical semantics.

The question then becomes how to represent the underspecification, and how to allow modifiers to manipulate it. The crucial step has been with us all along. The foundation of the paths approach to range expression has been the **start** and **end** predicates. We have never actually been explicit about whether a path that starts at an element must necessarily include that element, and likewise for where it ends. The facts here suggest that this is how it should remain. These predicates don't presuppose that paths must include their start points, so paths that start just after their start point are perfectly possible.

This also suggests a direction for adverbial endpoint modifiers. When a speaker asks that a range expression be construed inclusively, they normally intend just that its start- and endpoints be included in the range. That can be expressed naturally in the language of paths:

(99) a. $[\![\, \textit{inclusive(ly)} \,]\!] = \lambda p \,.\, \mathbf{start}(p) \in p \wedge \mathbf{end}(p) \in p$

b. $[\![\, \textit{exclusive(ly)} \,]\!] = \lambda p \,.\, \mathbf{start}(p) \notin p \wedge \mathbf{end}(p) \notin p$

These elements are simply properties of paths, inclusive or exclusive. They can be interpreted intersectively:

(100) $[\![\, \textit{five to ten exclusively} \,]\!] = \lambda p_p \,.\, \mathbf{start}(p) = \mathbf{5} \wedge \mathbf{end}(p) = \mathbf{10} \wedge \mathbf{start}(p) \notin p \wedge \mathbf{end}(p) \notin p$

The path-modifying variant of *between* would work similarly.

4.8 *Ignorance inferences*

A brief word about ignorance inferences. These are inferences that a speaker doesn't know a particular numerical value exactly, and is using a range expression or modified numeral to signal the fact:

(101) $\text{Floyd saw} \left\{ \begin{array}{l} \text{at least 3} \\ \text{at most 3} \\ \text{maximally 3} \\ \text{up to 3} \\ \text{3 to 6} \end{array} \right\} \text{capybaras.}$

This topic has occupied the modified numerals literature extensively (Büring 2007, Ciardelli et al. 2018, Coppock & Brochhagen 2013, Cremers et al.

2021, Kennedy 2015, Mayr 2013, Rett 2014b, Schwarz 2016a,b, Westera & Brasoveanu 2014), and its interactions with range expressions were a major focus of an earlier generation of this work (**?**). We will for the most part set the issue aside here, because the other empirical properties of range expressions this paper has focused on so far are probably more revealing about how range expressions themselves work. Nevertheless, it's worth highlighting one potentially interesting point about how ignorance inferences interact with the inclusive/exclusive distinction raised in the previous section.

One of the essential puzzles about ignorance inferences is why some expressions give rise to them and others don't seem to. The modified numerals in (101) do, but the ones in (100) don't:

(102) Floyd saw $\left\{\begin{array}{l}\text{fewer than 3}\\ \text{more than 3}\\ \text{less than 3}\end{array}\right\}$ capybaras.

If the speaker had asserted that they know exactly how many capybaras Floyd saw, it would be odd to go on to say (101) but not (102).

A leading idea about the source of the distinction (Kennedy 2015) is that it has to do with whether the ordering relation involved is strict ($>$ or $<$) or not ($\leq$ or $\geq$). Strict orders don't systematically give rise to ignorance inferences, the observation goes, but non-strict ones do. This is said to follow ultimately from Gricean inferences and a particular conception of what alternatives these inferences compare.

It is sometimes said that range expressions fit into this picture. *From/to* ranges are said to include their endpoints, which predicts—given this theory—that they shouldn't give rise to ignorance inferences. *Between/and* ranges are said to exclude them, which predicts that they should. Given the observations in section 4.7 immediately above, though, the picture changes. If range expressions are in general underspecified with respect to this distinction, it's less clear where ignorance readings should arise. More important, though, is that endpoint modifiers like *inclusive* and *exclusive* explicitly articulate precisely what's at issue here—strict vs non-strict—and therefore should be a straightforward means of turning ignorance inferences on or off. It's not obvious that this predicted behavior is found:

(103) I know exactly how many capybaras Floyd saw. It was $\left\{\begin{array}{l}\text{from 3 to 6}\\ \text{between 3 and 6}\end{array}\right\} \left\{\begin{array}{l}\text{inclusive(ly)}\\ \text{exclusive(ly)}\end{array}\right\}$.

The judgment is not perfectly clear, but without the endpoint modifiers *between* is probably more natural here, as predicted. But it's not at all clear that that is affected by the endpoint modifiers. The expectation is *inclusively* should be odd here because it conveys ignorance incompatible with the preceding sentence.

5 Concluding remark

The aim here has been to provide a description of the behavior of range expressions in English and an analysis that derives their particular constellation of properties ultimately from the locative foundation on which their semantics is built. More precisely, we proposed an analysis of their locative uses based in notion of paths, an idea amply independently motivated in the locatives literature. Generalizing paths across semantic types, it was possible to explain their uses both in degree and cardinality contexts. Interestingly, doing this offers additional evidence for the idea that both cardinal and ordinary measure phrases include an independent component of maximality. To implement this analysis, we relied on a type shift that relates degrees and paths in a simple but particular way, and the idea that range expressions are scope-bearing. The same approach sheds light on individual-based paths as well, and it may lay the groundwork for the explanation of how range expressions operate across syntactic categories more broadly still. Addressing that in earnest, though, will need to await future work.

Early on, we noted that range expressions have three kinds of readings: singleton punctual, set punctual, and interval. The paths approach has also yielded an explanation for all three. Singleton punctual readings, in which as single value is crucial, arise straightforwardly in the absence of appropriate quantification. Set punctual readings arise automatically when appropriate quantifiers scope over range expressions. Interval readings are associated with when a path is related to a contextually-provided superpath in the right way, without excluding any points within the crucial range.

The focus here has been resolutely on English, in part because it provides more than enough empirical richness to suffice for a single paper, but range expressions are common across the languages of the world, including in some languages in which the mathematical vocabulary has been systematically impoverished by a history of English-only schooling. The case in point is Ktunaxa, an isolate spoken in British Columbia and parts of the American northwest, where Sandoval (2024) shows ranges can be expressed via an interesting combination of conjunction and locative demonstratives. Many other languages use prepositional strategies, as English does, but there’s every reason to expect some interesting diversity in this domain.

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