

BOOK CHAPTER DRAFT

Vagueness, Degrees, and Gradable Predicates

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This is a draft of a chapter for a book, *Modification*, in preparation for the Cambridge University Press series *Key Topics in Semantics and Pragmatics*. The [full manuscript](#) is also available as a single document on [my website](#), as are some [additional chapters](#). The book is something between a textbook for people who already have a basic background in semantics and a survey of work in the area. For a fuller explanation of its purpose and scope, consult chapter 1 in the full manuscript.

Broken links, marked with ??, are to other chapters not included in this document. (You can avoid this by looking at the full manuscript.) **Comments would be extremely helpful**, so please don't hesitate to [contact me](#) if you have any, even very minor ones.

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1 Introduction

On a long car trip, one eventually encounters signs that say things like ‘now entering Massachusetts’. That seems reasonable enough. Sometimes, though, one encounters signs that say things like ‘scenic area’. This always struck me as faintly absurd. A government is perfectly entitled to draw lines on a map that define the precise boundaries between Massachusetts and adjacent states. But ‘scenic’? Has a transportation department employee been dispatched to discern the precise boundaries within which things have become—officially and legally—scenic? Why not also erect signs that say ‘ugly area’ or ‘disappointing region’ or ‘suburban sprawl’?

There are two linguistic issues that give rise to the sense of absurdity. One is important, but won’t be our concern in this chapter: the subjective quality of adjectives like *scenic* that’s incompatible with governments taking a position on them (see section ??). The other, however, is an aspect of a much larger question to which we will now turn. Even if we as a society decided to

delegate our aesthetic judgments to regional transportation authorities, we would still find it odd for them to draw fixed borders between what's scenic and what isn't. Being scenic—like being ugly, disappointing, or suburban—is an inherently incremental notion.¹

Such vagueness is an essential design feature of language. From a certain perspective, it is a problematic one. Formal semantics is founded on truth and falsity, binary notions that might seem to leave no wiggle room for the incremental. Yet in using language, we handle vagueness with aplomb. Sometimes, we cope with it by simply eliminating it: Clyde, we might say, is not merely *tall*, but *six feet tall* or *taller than Floyd*. At other times, we instead modulate the vagueness, and assert that he is, for example, *a little too tall*. We do this with concrete grammatical tools, morphemes we can identify and subject to analytical scrutiny.

Broadly construed, this will be the task of this chapter and the next. The initial challenge is how to reconcile the incrementality of vagueness with the discreteness of truth conditions. That's only the first step. Examining the grammar of vague predicates turns out to shed light not just on vagueness itself and its conceptual cousin, gradability, but also on the underlying structure of adjective meaning, the role of notions like 'scale' and 'dimension' in the grammar, and the nature of the constructions and expressions that specialize in manipulating this sort of meaning.

The chapter begins with a discussion of vagueness in section 2. Section 3 gives a thumbnail sketch of theories of vagueness and gradability and explores one approach that hasn't much captured the imagination of formal semanticists. Sections 4 and 5 presents two approaches that have. Section 6 compares them, considering what role the notion of 'degree' should have in the grammar. Finally, section 7 turns to scalar issues in the lexical semantics of adjectives.

2 Vagueness

2.1 Identifying vagueness

The first question to ask about vagueness is just what it is, precisely. When does a predicate count as vague? Perhaps the best answer is itself a kind of question. A predicate is vague if it gives rise to some version of the SORITES PARADOX ([sə'raɪtɪz]), the paradox of the heap ('sorites' being transliterated

¹A parallel example: the US National Weather Service sometimes refers in forecasts to 'bitter cold'. How severe must the cold be to be judged bitter? According to a number of vaguely official-looking charts online, from -19F to 0F, or -28C to -18C (<http://oceanservice.noaa.gov/education/yos/resource/JetStream/global/chill.htm>). Bitterness dissipates at lower temperatures, which are 'extremely cold'.

Greek for ‘heap’). In its canonical form, it begins with a heap of sand. If we remove a single grain of sand from the heap, we still have a heap. If we remove another, again, the heap remains a heap. In fact, we are typically willing to commit to a general principle: removing a single grain of sand is never enough to turn the heap into something other than a heap. Yet if we repeat this process, we will ultimately wind up with a single grain of sand, which surely isn’t a heap. But when did we lose the heap? Even in hindsight, we wouldn’t be comfortable identifying the crucial grain that moved us over the threshold between heap and non-heap. So there is a paradox: removing a single grain can never eliminate the heap, and yet the heap is gone. It works in reverse, too: begin with a grain of sand. You have no heap. Adding a single additional grain is never enough to create a heap. Yet do this repeatedly, and sooner or later, voilà: heap.

Of course, not all vagueness is about sand. But analogues are easy to dream up for arbitrary vague predicates. Another standard example is *bald*. Floyd, who is balding, isn’t actually bald. But what if we plucked a single hair from his head? And then another, and another? He will be, well, above all increasingly irritated, but also at some point bald. The crucial sequence of steps—removing grains of sand or hairs on his head—is called a *SORITES SEQUENCE*. Vague predicates systematically permit constructing one.²

Another way to look at the issue is in terms of *BORDERLINE CASES*. These are the points in a sorites sequence when our judgments begin to waver. There are individuals we consider bald and others we consider not bald, but there are some, the balding, that occupy an uncomfortable middle ground between bald and not bald. In some contexts and for some purposes, we might consider them bald, and in others not. Such borderline cases are another hallmark of vague predicates.

Vagueness is ubiquitous. It’s in obvious places, such as in the semantics of *GRADABLE ADJECTIVES*—that is, adjectives that admit degree modification or can occur in comparatives and related constructions. Accordingly, that’s the spot linguists have most concentrated on. But as the classic form of the sorites reflects, it’s also found in nouns as well. One can construct sorites sequences for PPs like *across the quad*.³ Verbs can be vague: *love* gives rise to

²The indifference we feel to the small changes that characterize each step in the sequence is termed *TOLERANCE*.

³The vagueness is even clearer with *near*, *close*, and *far*, which are sometimes mentioned as vague prepositions. But these may actually be adjectives, as their ability to occur as complements to *seem* and to form comparatives suggests:

- (i) He seemed $\left\{ \begin{array}{l} \text{nearer} \\ \text{closer to} \\ \text{farther from} \end{array} \right\}$ the quad.

borderline cases (for some people, apparently with alarming regularity), as can *run* (how fast must you go to count?). Indeed, it's not just vagueness that is ubiquitous. Its cousin, linguistic gradability is too (as Sapir 1944 and many subsequently have observed). As important as this is, some caution may be warranted. Gradability we can presumably take at face value, but certain instances of apparent vagueness might be better classified as imprecision (see section 2.3).

2.2 *Vagueness vs. ambiguity*

When I was about about five years old growing up in Poland, I found myself confused about the metaphysical status of Montreal. I had been told with categorical certainty that it's in America, and with equal certainty that it isn't. I was baffled. What sort of magical fairy-tale city was this 'Montreal', that it could be and—at the same time—not be in a particular place? My mistake, of course, was failing to recognize that *America* can refer either to the New World as a whole or to the United States in particular. It is only AMBIGUITY, and not vagueness, that can lead an innocent child astray in this way. *The western part of North America*, for example, is vague. Is Manitoba in the western part of North America? It's unclear. Manitoba is a borderline case. But this would never lead a child to mistake Manitoba for an Alice-in-Wonderland enigma, because no one would confidently assert that Manitoba both is and is not in the western part of North America. A lack of confidence of this sort is the defining feature of borderline cases, and it contrasts sharply with the bewildering certainty I encountered about Montreal.

This encapsulates the essential difference between vagueness and ambiguity. An ambiguous linguistic expression has more than one distinct interpretation, and the interpretations are discrete and one can enumerate them. They don't give rise to borderline cases, at least not without an independent source of vagueness. Some instances of ambiguity involve two distinct words that happen to be homophonous (lexical ambiguity): *bank* can mean either 'side of a river' or 'financial institution'. Other instances arise due to multiple syntactic structures that lead to multiple semantic representations (structural ambiguity). Thus the shopworn introductory-linguistics example *We saw the deer with binoculars* has readings in which either we or the deer have binoculars, depending on whether *deer with binoculars* is a constituent. The distinction between vagueness and ambiguity is not always straightforward, and we should be cautious about relying too much on such purely descriptive terminology without first committing to an explicit theory with respect to which it can be defined. But there are some useful tests that can jog one's intuitions about the difference (see Zwicky & Sadock 1975, Martin 1982, Zwicky & Sadock 1984, Gillon 1990).

The most straightforward of these is simply denying one reading while asserting the other (an example of this general strategy can be found in section ?? and, inadvertently, in the use of *America* described above). One can do this with the deer example straightforwardly. In a normal deer-viewing scenario, the sentence is true on the reading in which we have binoculars and false on the reading in which the deer has them. We could therefore truthfully both assert and deny that that string of words characterizes the situation. Another angle on the same idea is finding a scenario that would render the sentence true on one reading and false on the other, and a distinct scenario that renders the previously true reading false and the previously false reading true. Thus in the deer example, the situation is reversed in a scenario in which we saw the deer unaided while the deer had binoculars.

A more subtle and interesting tool is ZEUGMA ([ˈzʊgmə]),⁴ the use of a word in two different senses simultaneously. It gives rise to a characteristic sense of anomaly that is absent in superficially similar non-zeugmatic structures. Suppose that we were interested in determining whether *expired* is lexically ambiguous between the meaning ‘became out of date’ and ‘became dead’, or whether it’s a general cover term for both. The zeugmatic example in (1), due to Cruse (1986), might settle the question:

(1) ?John and his driving license expired yesterday.

This predicates *expired* of *John* and *his driving license* in different senses. It feels odd—like a kind of half-joke—because *expired* is, indeed, ambiguous. In the absence of ambiguity, the odd feeling is absent too. Car doors are very different from house doors, and one might suspect that the word *door* is ambiguous between these two senses. But it isn’t, as the lack of a zeugmatic flavor in (2) attests:⁵

(2) Floyd’s house and car both had a broken door.

As it turns out, *door* has a single meaning that encompasses both. One might say that *door* is therefore vague with respect to this distinction, though it’s

⁴A related term is ‘syllepsis’, apparently used mostly by literary scholars rather than linguists (Pinkal 1995 mentions it in scare quotes). It doesn’t seem to have a consistently-observed definition sufficiently rigorous to distinguish it from zeugma in way that is a grammatically principled and analytically useful.

⁵This contrasts with, for example:

(i) ?Floyd and his car both had $\left\{ \begin{array}{l} \text{an uncomfortable seat} \\ \text{a short fuse} \end{array} \right\}$.

Sexist examples involving *rack* could also be constructed.

not actually self-evident that this is quite the same as the notion of vagueness we've examined so far. In a way, it doesn't matter. What the zeugma test shows is just that this isn't an ambiguity. Useful though the test is, it does depend precariously on the sensation of 'oddness'. Many things make us feel odd, only some of which are relevant here. Caveat emptor.

Before moving on, it's worth highlighting another form of indeterminacy: POLYSEMY. This describes the state of affairs in which a word has multiple senses that are clearly related. A *big country*, for example, can be one with a large population or a large landmass. A *bed* is normally what one sleeps on, but this meaning is related to uses like *a riverbed* or *a bed of lettuce*. It's not always clear when an ambiguity should be called a polysemy. From the usual perspective of formal semantics, nothing hinges on the difference: the typical analytical strategy is to simply treat polysemy as a special case of lexical ambiguity. Whether we should be embarrassed by this or proud of it is itself a little unclear. But there is certainly formally explicit work that wrestles with the issue without writing it off. It includes Nunberg (1995), Pustejovsky (1995), Pustejovsky & Bouillon (1995), Lascarides et al. (1996), Blutner (1998), Alrenga (2007), Blutner (2008), Brasoveanu (2008), Katz (2008).

It's also not clear whether the term 'polysemy' groups together a natural class of phenomena. The distinct interpretations of *skillful* in *a skillful surgeon* and *a skillful thief* or of *beautiful* on the two readings of *beautiful dancer* (see section ?? and indeed much of chapter ?? generally) might be termed polysemy, yet they are rather different from the distinct meanings of *bed*. Moreover, in *beautiful dancer*, the two readings might arise—depending on the analysis—as the result of a structural ambiguity, or else as the result of an implicit argument that can receive distinct values from the discourse context. On the latter analysis, the context-dependence renders this more similar to vagueness than ambiguity after all.

This reinforces the conclusion that one shouldn't put too much stock in terminology. What matters is the analysis, and until we provide a sufficiently explicit one, we can't render a verdict about what semantic phenomenon is at issue.

2.3 *Vagueness vs. imprecision*

Vagueness, as we've seen, is distinguished by borderline cases, cases for which we are hesitant—however much we might be pressed—to assign a truth value. There is a superficially similar phenomenon that doesn't behave this way. It's reflected in the contrast between (3a) and (3b):

- (3) a. Floyd is tall. (vague)
 b. Floyd is six feet tall. (potentially imprecise)

To make things explicit, suppose (3b) is true, and Floyd is in fact six feet tall. Without further information, we don't know whether to judge (3a) true or false. This run-of-the-mill vagueness is completely resolved in (3b). We require no further information to judge (3) true or false. We know precisely what it would take for Floyd to be six feet tall. If we could shrink Floyd in tiny steps, we could construct a sorites sequence for (3a), but not for (3b). We would always in principle be able to identify the exact step that made him less than six feet tall.

The difference is apparent in everyday use. No reasonable person would agree to a bet that is to be determined on the basis of whether some as-yet-unseen individual turns out to be tall. There would be no objective way to resolve such a bet. This uncertainty contrasts starkly with how we react to sentences such as (3b). A perfectly reasonable person might place a bet on whether someone turns out to be six feet tall. The MEASURE PHRASE *six feet* eliminates vagueness.

This seems relatively straight-forward. But in some unusual circumstances, the situation becomes murkier. If, for example, I have agreed to the bet that the unseen person is six feet tall, I might still find myself in an argument once this person—Floyd—has presented himself and agreed to be measured. It might turn out that Floyd is just barely shorter than six feet, by a tiny fraction of an inch. Here, again, there seems to be a kind of uncertainty. Yet this uncertainty is of a quite different kind. If we have established conclusively that Floyd's height falls short of six feet, even by a fleetingly small amount, it would be very difficult for me to insist that he is nevertheless six feet tall. With this information, I could convince no one that (3b) is true. Any argument that breaks out is not about the truth value of (3b) as such. It is rather about how precisely we want to interpret the terms of our bet. To weasel out of the bet, I might accuse you of being unreasonable or pedantic in insisting that (3b) is false, but I could not accuse you of being wrong about it. So, despite the dispute, (3b) does not seem to be vague. It is, however, potentially IMPRECISE, and we can disagree about the intended level of precision.

Taken to its logical conclusion, this all has the odd consequence that, speaking absolutely strictly, it is improbable that anyone is (exactly) six feet tall. With sufficiently precise instrumentation, we would discover that virtually everyone falls at least an atom or two short, or is at least an atom or two too tall. This is sheer pedantry, of course—but again, it is not wrong. This is the insight that Lasersohn (1999) articulates in especially clear terms. In ordinary use, we are happy to judge true sentences that, if really pressed, we would be forced to admit are technically false. We allow ourselves what he called PRAGMATIC SLACK. Imprecision is at heart not an issue of truth or falsity as such, but of how close an approximation of truth is pragmatically sufficient in a particular context.

Seem is sensitive to this distinction. It is compatible with vague predicates (Matushansky 2002), but not with ones that are merely imprecise:

(4) Clyde seems (*six feet) tall.

The amount of pragmatic slack speakers give each other is of course not typically made explicit, but a variety of linguistic devices can help make it clear. *Precisely*, for example, restricts the amount of pragmatic slack available:

(5) Floyd is precisely six feet tall.

For (5) to be judged close enough to true, Floyd has to be closer to being six feet tall than if *precisely* were absent.

This distinction between these two flavors of linguistic uncertainty—vagueness and imprecision—is to be found in various forms and with various labels in Pinkal (1995), Lasersohn (1999), Kennedy & McNally (2005), Kennedy (2007), Sauerland & Stateva (2007, 2011), Morzycki (2011), van Rooij (2011), Bouchard (2012), Klecha (2013), and Anderson (2013, to appear). Sometimes one of these notions is taken to include the other, as in Lewis (1979), who explicitly invokes ‘standards of precision’ and views it as a kind of vagueness. Sauerland & Stateva make a case for preferring the terms ‘scalar vagueness’ and ‘epistemic vagueness’ for (ordinary) vagueness and imprecision, respectively, but provide further evidence for a distinction. The variation of views is an indication that the distinction between vagueness and imprecision isn’t an obvious one, and the issue of how best to think about it remains unsettled. Further work will, I hope, help clarify the situation.

Lasersohn (1999) conceptualizes imprecision in terms of PRAGMATIC HALOS. The pragmatic halo of an expression is a set of objects of the same type as its denotation which differ in only ‘pragmatically ignorable’ ways. Thus, in most contexts, $\llbracket \textit{six feet} \rrbracket$ has a halo around it consisting of lengths that are near enough to six feet not to make any difference: $5'11\frac{1}{2}''$ – $6'\frac{1}{2}''$, say. Halos expand compositionally. The halo of $\llbracket \textit{six feet long and three feet wide} \rrbracket$ combines the halos of $\llbracket \textit{six feet tall} \rrbracket$ and $\llbracket \textit{three feet wide} \rrbracket$, so that it might include objects that are $5'11\frac{1}{2}''$ tall and $2'11\frac{1}{2}''$ wide. Interestingly, to provide a semantics in this spirit for slack-regulators like *precisely* (see also section ??), it’s necessary for the compositional semantics to gain access to—and in that sense, to be interleaved with—the machinery by which haloes are generated. Pragmatic haloes, evidently, are not purely pragmatic.

2.4 Some foundational questions

In the discussion that follows, I will sidestep a number of interesting philosophical issues about vagueness in order to focus on the ones most directly relevant to linguistic semantics. Nevertheless, it's worth at least raising some foundational questions about the origin of vagueness.

One of these is whether vagueness is a property of linguistic expressions or objects in the world. We might be uncertain about the precise point at which the Sahara desert starts. Does that mean the proper name *the Sahara* is vague? Or is it the desert itself that is vague? The latter possibility is called ONTOLOGICAL VAGUENESS. The notion is rejected in [Russell \(1923\)](#) and [Evans \(1978\)](#), but has its defenders (see [Sorensen 2012](#)).

Even if we grant that it is language that is the locus of vagueness, we don't need to assume that it arises from an inherent indeterminacy in linguistic expressions. There is another way of thinking about it, due to [Williamson \(1994\)](#): vagueness might arise instead from our ignorance as speakers. On this view, it's not that some things are inherently incremental, but that our knowledge of them is inherently incomplete. The solution to the sorites paradox, then, is to reject the premise that removing a single grain of sand can't on its own eliminate a heap. There is simply a fact of the matter that is hidden from us: that a heap must have (let's suppose) 100,042 grains of sand. Reducing that number by one makes it no longer a heap. That we don't **know** this fact doesn't make it any less a fact. No one knows the precise number of grains of sand on Earth either, yet we don't dispute that there is a fact of the matter there. We just confess that we don't know (or care) what it is. Why, then, should we not take the same attitude to heaps?

At first, this idea—dubbed the EPISTEMIC VIEW of vagueness—seems counterintuitive, perhaps partly because the ubiquity of vagueness would entail a corresponding ubiquity of ignorance. But to paraphrase H.L. Mencken, no one ever lost money by overestimating human ignorance.⁶ I think I know what *dead* means and generally have no hesitation in distinguishing the living from the dead. Yet there are tragic apparent borderline cases, and in those, I defer to doctors. (See [Putnam 1975](#) for the classic argument that an individual speaker might have only partial knowledge of meaning in this way.) More or less similarly, I think I know what *winner* means in the context of an election. Yet if told the vote count of a candidate but not the number of candidates or total votes cast, I couldn't determine whether she is the winner, or even the proportion of votes necessary to win. Despite this ignorance, I wouldn't be tempted to conclude that there is therefore no determinate

⁶The original quote is 'no one . . . has ever lost money by underestimating the intelligence of the great masses of the plain people' (1926; 'Notes on Journalism'; *Chicago Tribune*).

winner or no specific vote quota the winner must reach. As a first step, it's enough to confess that other predicates might be roughly similar to these.

Williamson takes an important further step. In general, there are no authorities to tell us where sharp boundaries lie. A word on this view is defined instead by an unspoken and unconscious consensus among native speakers, so at any given moment, its meaning—and therefore the location of sharp boundaries—depends on a general patterns of use no individual speaker can track precisely. For most vague predicates, then, it is not just that we don't know where the boundaries lie. It's that we **can't** know. At a stroke, this would resolve the tension between vagueness and the methodological assumption that there are only two truth values.

All that said, we will proceed on the assumption that an account of the semantics of natural language must include an account of (at least some) vagueness.

3 Theories of vagueness and gradability: a false start

3.1 *Three approaches*

The literature contains various claims of the form 'there are n principal (classes of) theories of vagueness', where n varies. They are then enumerated in a way that adheres to certain general conventions but otherwise also varies. I mention this so that the reader will approach the next paragraph in the right spirit.

There are three principal classes of theories of vagueness. They are:

- FUZZY-LOGIC THEORIES, in which there is a scale consisting of infinitely many truth values. They have not played a major role in formal linguistic semantics, except perhaps as a foil.
- What I'll call INHERENT VAGUENESS THEORIES, which are often referred to with terms including 'supervaluation', 'delineation', and 'extension gap'. In these theories, certain sentences with vague predicates may lack a truth value and there is no direct representation of measurement.
- DEGREE-BASED THEORIES, which introduce objects into the model called 'degrees' to directly represent measurement and assume these objects can serve as arguments to gradable predicates (or, alternatively, can be what gradable predicates yield in place of a truth value).

Importantly, this lists semantic frameworks for analyzing vagueness and gradability rather than general views of what vagueness is (like, for example, the epistemic theory of vagueness, or the idea that vagueness stems from context-sensitivity).

3.2 Fuzzy logic

There is certainly a *prima facie* tension between the incremental quality of gradable predicates and the idea that there are only two truth values. Moreover, the idea that sentences must be simply true or false seems to fly in the face of common sense. It's hard to imagine a more banal truism than that life is not in black and white but full of shades of gray. So why should the semantics be founded on something so deeply unintuitive?

The answer emerges in considering what the alternative would look like. The most dangerously seductive option is to allow an infinite number of truth values, including all real numbers between 0 and 1. One could be more cautious and simply introduce a single additional truth value, but that doesn't scratch the relevant analytical itch—life is full of **shades** of gray, not full of black, white, and gray. Embracing infinitely many truth values is what distinguishes FUZZY LOGIC from classical logic (Zadeh 1965, 1983; Pelletier 2000 points out that infinite-valued logics without the catchy name date to Łukasiewicz 1920).

Fuzzy logic has for the most part not played a prominent role in formal linguistics (a notable exception is Lakoff 1973), so I will only sketch some of the difficulties it presents. (See Kamp 1975, Fine 1975, and Kamp & Partee 1995 for further discussion.) First, it makes odd predictions about the truth values of coordinated sentences such as (6):

- (6) a. Floyd is tall or he isn't tall.
- b. Floyd is tall and he isn't tall.

Intuitively, if Floyd is a borderline case for *tall*, we would probably want these to have different truth values: (6a) is true, and (6b) false.⁷ On at least one natural implementation of fuzzy logic, however, that's not what would happen. Fuzzy connectives could be defined as in (7) (Zadeh 1965, Kamp & Partee 1995; I've translated the set-theoretic characterization into its logical counterpart):

- (7) a. $\llbracket \textit{not } \phi \rrbracket = 1 - \llbracket \phi \rrbracket$
- b. $\llbracket \phi \textit{ and } \psi \rrbracket = \text{the lower of these truth values: } \llbracket \phi \rrbracket, \llbracket \psi \rrbracket$
- c. $\llbracket \phi \textit{ or } \psi \rrbracket = \text{the higher of these truth values: } \llbracket \phi \rrbracket, \llbracket \psi \rrbracket$

⁷The latter judgment might be clouded by linguistic conventions like saying 'Is he tall? Well, he is and he isn't, depending.' The cloudiness may be dispelled with various rewordings:

- (i) a. It's true that Floyd is tall and it's true that he isn't.
- b. It is and is not the case that Floyd is tall.
- c. It's true that Floyd is tall and it's false that Floyd is tall.

The idea behind (7a) is that the negation of a proposition is exactly as true as the original proposition was false. If it's mostly true that I'm tall, it's mostly false that I'm not. What (7b) reflects is that conjoined proposition is as true as its least true conjunct: the claim that I'm tall and triangles have four sides is simply false, no matter my height. What (7c) reflects is the corresponding fact about disjunction: the claim that I'm tall or triangles have four sides is precisely as true as it is that I'm tall. With this in mind, suppose *Floyd is tall* has a truth value of 0.5. Following these rules, the same truth value, 0.5, would be assigned to its negation, *Floyd isn't tall*, and therefore also to both (6a) and (6b). Bad news.

Second, one of the things one might want from a theory of vagueness is some insight into gradability and comparison. On a fuzzy logic approach, comparatives might be interpreted by comparing truth values directly:

- (8) a. Floyd is taller than Clyde.
 b. $\llbracket \textit{Floyd is tall.} \rrbracket > \llbracket \textit{Clyde is tall.} \rrbracket$

There is something uncomfortable about this. Judgments about relative height feel subjectively very different from judgments about relative truth. There is a world of difference between asserting (8) and asserting, for example, that it would be more of a lie to claim that Clyde is tall than it would to claim that Floyd is tall. Perhaps these subjective impressions are misleading, and there is a way of disentangling them from the fuzzy machinery. But they require some explanation—and it doesn't bode well for the approach when it must explain away grammatical intuitions right from the outset. Moreover, as [Nouwen et al. \(2011\)](#) point out, putting all comparatives on the same scale—that of truth values—means it should be possible to interpret comparatives composed of arbitrary pairs of sentences, as in (9):

- (9) a. #Floyd is taller than this is a ferret.
 b. #Floyd is taller than Clyde is unpleasant.

Perhaps (9a) can be ruled on out purely syntactic grounds. That seems unsatisfying, since it feels like something has gone wrong semantically too in a way we might want the semantics to reflect. But even if we were to set it aside, this mode of explanation is unavailable for (9b), which is syntactically pristine. Yet of course, tallness and unpleasantness manifest a fundamental INCOMMENSURABILITY. Setting aside some complications (see sections ?? and ??), comparatives built around unrelated properties are systematically ill-formed in just this way. Having a single scale might be desirable for certain purposes ([Bale 2008, 2006](#)), but this isn't the right way to achieve it.

4 The inherent vagueness approach

4.1 *Extension gaps*

If we must resist the siren song of fuzzy truth values, it would be nice to hold on at least to the intuition that for borderline cases, vague predicates are neither true nor false. It turns out that we can. On a standard semantics, a predicate like *tall* has as its extension the set of tall people. In a slight terminological modification, we could call this the POSITIVE EXTENSION of *tall*, and call everything that isn't tall its NEGATIVE EXTENSION. This would suffice in a world of absolutes, but we'd like to find a place for the borderline cases. These, it might be said, fall into an EXTENSION GAP: the set of things in neither the positive nor the negative extension of *tall*; that is, the set of things of which it is neither true nor false. Correspondingly, sentences in which a vague predicate is predicated of a borderline case fall into a TRUTH-VALUE GAP.

This idea lies at the heart of one class of approaches (Fine 1975, Kamp 1975, and Klein 1980, 1982 develop the core framework, building on formal tools in van Fraassen 1966; work broadly in this spirit includes Lewis 1970, McConnell-Ginet 1973, Ballweg 1983, Pinkal 1983, Larson 1988, Kamp & Partee 1995, Sassoon 2013a, 2007, 2013b, 2010b, van Rooij 2008, Krasikova 2009, Doetjes 2010, Doetjes et al. 2011). The names by which people refer to these theories—or various subsets of them—aren't consistent, and generally involve picking the name of a certain component of the theory as a name for the whole. These include 'supervaluation(ist)', 'delineation', 'comparison class', 'precisification', 'vague predicate' (with 'theory' or 'approach' appended, of course).⁸ Depending on the author, these might not pick out precisely the same class of ideas, but they are part of the same broad intellectual current. At the risk of compounding the problem, I'll refer to them by yet another term, as 'inherent vagueness' theories, to reflect that they build on the intuition that vagueness is inherent in vague predicates themselves rather

⁸The term 'delineation theory' has a catchy ring, but it strikes me as non-optimal. 'Delineation' refers to the sharpening of a vague predicate by eliminating borderline cases, a cut-off used in doing so, or a function associated with either. The term hardly occurs in work that established the approach—none of van Fraassen (1966), Fine (1975), Kamp (1975), Klein (1980, 1982), Ballweg (1983), Pinkal (1983), Larson (1988) or Kamp & Partee (1995) use it. (The sole exception is McConnell-Ginet 1973.) More important, both the concept and the term are no less at home in degree-based theories. Barker (2002), for example, explicitly treats delineations as functions that map a gradable predicate to a degree, which is fully consistent with the (apparently) original use of the term Lewis (1972). Setting aside independent differences, this is also what the standard-determining predicate does in the degree-based analysis of Kennedy (2007)—and, indeed, in this book. For similar reasons, 'the comparison class approach' doesn't seem optimal either.

than—as on a degree-based theory—the result of how they enter into the compositional semantics.⁹

What I'll present here is a version of the idea mostly in the spirit of Klein (1980, 1982). First, we need to introduce extension gaps into the system. This can be done by assuming that vague predicates denote partial functions, ones that are simply undefined for individuals in their extension gap. The most familiar use of partial functions in natural language semantics is as a means of representing presuppositions, so this represents a departure. (Though it's worth reflecting on what similarities to presupposition there might be.)

A second crucial component is discourse contexts. What counts as a borderline case varies from one context to another.¹⁰ If we're discussing basketball players, many people we might ordinarily describe as tall would instead be borderline cases, and therefore in the extension gap of *tall*. If we're discussing children, many people that might otherwise be borderline cases would instead fall in the positive extension of *tall*. In one context, *tall* would mean something like 'tall for a basketball player', and in the other, 'tall for a child'. These sets of individuals—basketball players or children—are distinct COMPARISON CLASSES. They must be at least part of what a context supplies. (Beyond that, I will remain noncommittal on how contexts should be represented. Less Klein-influenced implementations of this approach use another formal tool, partial models, instead.)

With that in place, one can venture a denotation. The following predicates will be useful:

- (10) a. $\mathbf{gap}_c(P)(x) \stackrel{\text{def}}{=} 1$ iff x is in the extension gap of P in context c
b. $\mathbf{pos}_c(P)(x) \stackrel{\text{def}}{=} 1$ iff x is in the positive extension of P in context c
c. $\mathbf{neg}_c(P)(x) \stackrel{\text{def}}{=} 1$ iff x is in the negative extension of P in context c

I will use c as a variable for contexts, which will be introduced as an index on the interpretation function $\llbracket \cdot \rrbracket$. Thus:

⁹This is presumably the intuition behind the term 'vague predicate analysis' as well, but I avoid it because 'vague predicate' is a useful pre-theoretical descriptive term to describe predicates that are vague, without regard to how their vagueness is analyzed.

¹⁰This essential analytical impulse is an important aspect of what is referred to in the philosophical literature as CONTEXTUALISM. With respect to vagueness, this emphasis on contexts leads to a view of sorites sequences in which each step is a kind of incremental coercion of one context into another (Kamp 1981a). See Stanley (2003) for an ingenious and alarming counterargument to this from ellipsis, and van Rooij (2011) for a counterargument to the counterargument. It's worth pointing out that—in the particular versions presented here—this view of sorites sequences is in principle compatible with both inherent-vagueness and degree-based approaches.

$$(11) \quad \llbracket \textit{tall} \rrbracket^c = \lambda x : \neg \text{gap}_c(\textit{tall})(x) \cdot \text{pos}_c(\textit{tall})(x)$$

To yield a result, this function requires that the individual it applies to, x , must not be in the extension gap of **tall** in context c (using the colon notation for partial functions in Heim & Kratzer 1998). If this requirement is satisfied, the function will yield 1 if x is in the positive extension and 0 otherwise (i.e., if x is in the negative extension).

4.2 *Precisification and supertruth*

The next crucial component is the observation that contexts aren't static. As the discourse unfolds, old contexts are extended and elaborated into a new ones. One way this can happen is by the accumulation of information that allows the interlocutors to close in on a consensus about, for example, who counts as tall (see Barker 2002 for an especially direct implementation of this insight). Indeed, in some cases, one can even imagine interlocutors explicitly assigning various borderline cases to the positive and negative extension of *tall*: 'Clyde? He's tall-ish, but I wouldn't really say he's tall. As for Floyd, he's **definitely** tall.' One can even imagine, hypothetically, that the discourse could continue to the point that no extension gap remains. A context such as this—one in which the extension gap is empty—is called a total PRECISIFICATION (or a 'completion', or one that provides a 'delineation' of the predicate).¹¹

Precisifications play at least two key roles. One of them is that they keep us from running aground on the shores that doomed the fuzzy logic strategy. Its undoing was in part the failure to ensure that (12a) is always true and (12b) always false:

- (12) a. Floyd is tall or he isn't tall.
 b. Floyd is tall and he isn't tall.

To arrive at the right result, though, it will be necessary to nudge the notion of truth slightly from its customary place. If Floyd is a borderline case, these sentences both come out simply undefined. That's assuming the ordinary assignment of truth conditions to these sentences—that is, the ordinary VALUATION. But they are nevertheless special. On any total precisification, (12a) will come out true. If we assign Floyd to the positive extension of *tall*, the sentence will be true because of the first conjunct; if we assign him to the

¹¹For a context to count as a precisification of another, it must meet conditions including, informally, respecting orderings already present in the model. (A brief discussion follows in section 4.3; see Klein 1980, 1982 for details, or Kennedy 1997 for an especially accessible exposition.)

negative extension, because of the second. The assignment of truth conditions on the basis of all total precisifications is a SUPERVALUATION, and it renders a sentence such as (12a) SUPERTRUE. Precisely the same reasoning renders (12b) SUPERFALSE. And, as these examples demonstrate, it's supertruth and superfalseness that really count in reflecting our intuitions.

The notion of supertruth doesn't change the picture with respect to simple positive sentences. Before, *Floyd is tall* would come out undefined if Floyd is in the extension gap. That remains the case. But is it supertrue? It isn't true on all total precisifications, so no. Nor is it superfalse. If our intuitions about truth and falseness are sensitive to supertruth and superfalseness, these sentences would still come out neither true nor false.

4.3 Comparatives

The other major role for precisifications is in the semantics of comparatives. Again, assuming that Floyd is in the extension gap of *tall*, we'd still like (13) to be able to come out true:

(13) Floyd is taller than Clyde.

A similar move—quantifying over precisifications—will accomplish this. If Floyd is actually taller than Clyde, a total precisification could do one of three things (individuals are listed in descending order of height):

(14) a. assign both Floyd and Clyde to the positive extension:

Greta	}	<i>positive extension</i>
Floyd		
Clyde		
Herman	}	<i>negative extension</i>

b. assign both Floyd and Clyde to the negative extension:

Greta	}	<i>positive extension</i>
Floyd	}	<i>negative extension</i>
Clyde		
Herman		

- c. assign Floyd to the positive extension and Clyde to the negative extension:

Greta	}	<i>positive extension</i>
Floyd		
Clyde	}	<i>negative extension</i>
Herman		

What a precisification could **not** do is assign Clyde to the positive extension and Floyd to the negative one. There is no way of drawing the boundary between the tall and the not-tall that would count a taller person as not-tall and a shorter one as tall. But the crucial case is (14c). That precisification is only possible because Floyd is, in fact, taller than Clyde.

So we've arrived at the truth conditions of a comparative: *Floyd is taller than Clyde* is true iff there is a way of drawing the boundary between the tall and not-tall that leaves Floyd in the tall group and Clyde in the not-tall group. More precisely, it's true iff there is a precisification on which Floyd winds up in the positive extension and Clyde doesn't:

$$(15) \quad \llbracket \text{Floyd is taller than Clyde} \rrbracket^c \\ = \exists c' \in \text{precisifications}(c) \left[\begin{array}{l} \llbracket \text{tall} \rrbracket^{c'}(\text{Floyd}) \wedge \\ \neg \llbracket \text{tall} \rrbracket^{c'}(\text{Clyde}) \end{array} \right]$$

Where $\text{precisifications}(c)$ is the set of total precisifications of c . Because we're dealing with total precisifications, we can disregard the partiality of the functions (on a total precisification, they are defined for all individuals in the domain).¹² Thus (15) amounts to:

$$(16) \quad \llbracket \text{Floyd is taller than Clyde} \rrbracket^c \\ = \exists c' \in \text{precisifications}(c) \left[\begin{array}{l} \text{pos}_{c'}(\text{tall})(\text{Floyd}) \wedge \\ \neg \text{pos}_{c'}(\text{tall})(\text{Clyde}) \end{array} \right] \\ = \exists c' \in \text{precisifications}(c) \left[\begin{array}{l} \text{pos}_{c'}(\text{tall})(\text{Floyd}) \wedge \\ \text{neg}_{c'}(\text{tall})(\text{Clyde}) \end{array} \right]$$

The last step is licit because, when the extension gap is empty, if Clyde isn't in the positive extension he must be in the negative one, and vice versa. (That said, even on a partial precisification, effect of the comparative would be achieved even without having taken this last step.)

¹²The restriction to total precisifications here is only for conceptual clarity. The partiality of the function $\llbracket \text{tall} \rrbracket$ would be harmless even on a partial precisification.

Importantly, this correctly predicts that the comparative does not license inferences to the POSITIVE FORM of the adjective (the bare, morphologically unmarked form): we can't conclude from *Floyd is taller than Clyde* that he is tall. This follows because the precisifications stay resolutely inside the scope of the existential quantifier. With respect to the main, matrix context of evaluation, everything remains just as it was: the inhabitants of the positive and negative extensions and the extension gap are unchanged.

This approach—quantifying over something similar to potential continuations of the discourse context—anticipates a major mechanism in theories of dynamic semantics that would be developed later, such as Heim (1982), Kamp (1981b), and their many intellectual descendants. On such views, quantificational expressions (determiners, modals, adverbs) also quantify over ways of extending a discourse context. This isn't always apparent in work in the inherent-vagueness framework in part because the 'extending the context' language either wasn't used or wasn't emphasized until Pinkal (1995) and Barker (2002). An explicit connection isn't necessarily drawn between this account of the comparative and the dynamic treatment of quantifiers (more precisely, between quantifying over precisifications and quantifying over assignment functions). The nature of such connections, and if indeed there are any, is a question worth pondering.

But how to arrive at the desired truth conditions compositionally? The denotation of the comparative morpheme (*-er/more*) should be as in (17), where α is a gradable adjective:

$$(17) \quad \llbracket \text{more } \alpha \rrbracket^c = \lambda x \lambda y . \exists c' \in \mathbf{precisifications}(c) \left[\begin{array}{l} \llbracket \alpha \rrbracket^{c'}(x) \wedge \\ \neg \llbracket \alpha \rrbracket^{c'}(y) \end{array} \right]$$

This isn't a fully compositional denotation, though, because it fails to assign a denotation to *more* on its own, independent of the adjective. The difficulty is that *more* needs access to the context parameter of the adjective it combines with. If it simply gathered up an adjective meaning (type $\langle e, t \rangle$), it wouldn't get this access. What it actually needs is a function that it can feed precisified contexts into, something of type $\langle c, et \rangle$, where c is the type of contexts. One way of achieving this is with a new rule of semantic composition, a close cousin to ordinary functional application that stands in roughly the same relation to it as intensional functional application does:

(18) CONTEXT-ACCESSING FUNCTIONAL APPLICATION

If a branching node α has as its daughters β and γ , and $\llbracket \beta \rrbracket^c$ applies to expressions of type $\langle c, \dots \rangle$ and $\llbracket \gamma \rrbracket^c$ is of type $\langle \dots \rangle$, then

$$\llbracket \alpha \rrbracket^c = \llbracket \beta \rrbracket^c (\lambda c' . \llbracket \gamma \rrbracket^{c'})$$

Accessing a context index in this way may be useful in other analytical circumstances (Schlenker 2003). Indeed, Klein points out that functions from contexts to extensions are precisely what Kaplan (1989) proposed in his groundbreaking work on demonstratives. (In Kaplan’s terminology, a function of type $\langle c, \dots \rangle$ is the ‘character’ of a function of type $\langle \dots \rangle$.)

With this in place, a compositional denotation is possible (I assume *than* is not interpreted):

$$\begin{aligned}
 (19) \quad a. \llbracket \text{more} \rrbracket^c &= \lambda f_{\langle c, et \rangle} \lambda x \lambda y . \exists c' \in \mathbf{precisifications}(c) \\
 & \qquad \qquad \qquad [f(c')(x) \wedge \neg f(c')(y)] \\
 b. \llbracket \text{more tall} \rrbracket^c &= \llbracket \text{more} \rrbracket^c (\lambda c'' . \llbracket \text{tall} \rrbracket^{c''}) \\
 &= \lambda x \lambda y . \exists c' \in \mathbf{precisifications}(c) \\
 & \qquad \qquad \qquad [[\lambda c'' . \llbracket \text{tall} \rrbracket^{c''}](c')(x) \wedge \neg [\lambda c'' . \llbracket \text{tall} \rrbracket^{c''}](c')(y)] \\
 &= \lambda x \lambda y . \exists c' \in \mathbf{precisifications}(c) \\
 & \qquad \qquad \qquad [\llbracket \text{tall} \rrbracket^{c'}(x) \wedge \neg \llbracket \text{tall} \rrbracket^{c'}(y)] \\
 & \qquad \qquad \qquad [\mathbf{pos}_{c'}(\mathbf{tall})(x) \wedge \neg \mathbf{pos}_{c'}(\mathbf{tall})(x)] \\
 c. \llbracket \text{more tall than Clyde} \rrbracket^c &= \llbracket \text{more tall} \rrbracket^c (\llbracket \text{than Clyde} \rrbracket^c) \\
 &= \llbracket \text{more tall} \rrbracket^c (\mathbf{Clyde}) \\
 &= \lambda y . \exists c' \in \mathbf{precisifications}(c) \\
 & \qquad \qquad \qquad [\mathbf{pos}_{c'}(\mathbf{tall})(x) \wedge \neg \mathbf{pos}_{c'}(\mathbf{tall})(\mathbf{Clyde})]
 \end{aligned}$$

The syntax-semantics interface assumptions behind this structure—that *more tall* denotes a relation between individuals—certainly aren’t sufficient to account for the range of English comparatives, but they will suffice for now. Chapter ?? will present some more general, and therefore more sophisticated, options. This chapter will, for the sake simplicity, stubbornly persist in this mistake.

4.4 Degree words

A theory of gradability should include not just a means of understanding comparatives, but also degree words such as *very*. This approach offers that possibility.

Klein suggests that degree words like *very* have, sensibly enough, precisely the same type as the degree morpheme *more*: functions from characters to extensions, type $\langle \langle c, et \rangle, et \rangle$. This is natural, since they have the same syntactic category. Comparatives accomplish their work by quantifying over contexts, so one might expect *very* to do this too. The question, then, is what effect *very* has on the context with respect to which a gradable adjective is evaluated.

Klein’s answer is that it changes the comparison class. The gradable predicate is interpreted not with respect to the current context’s comparison class, but rather a comparison class that consists only of the members of its current positive extension. Someone who is *very tall* is ‘tall even compared to the people we’ve already established are tall’, or, more pithily, ‘tall (even) for a tall person’. The denotation and a first step in semantic composition are in (20) and (21):

- (20) a. $\llbracket \text{very} \rrbracket^c = \lambda f_{(c, et)} \lambda x . f(c')(x)$
 where c' is identical to c except that the comparison class in c' is $\{y : f(c)(y)\}$
- b. $\llbracket \text{very tall} \rrbracket^c = \llbracket \text{very} \rrbracket^c (\lambda c'' . \llbracket \text{tall} \rrbracket^{c''})$
 $= \lambda x . [\lambda c'' . \llbracket \text{tall} \rrbracket^{c''}](c')(x)$
 where c' is identical to c except that the comparison class in c' is $\{y : [\lambda c'' . \llbracket \text{tall} \rrbracket^{c''}](c)(y)\}$
 $= \lambda x . \llbracket \text{tall} \rrbracket^{c'}(x)$
 where c' is identical to c except that the comparison class in c' is $\{y : \llbracket \text{tall} \rrbracket^c(y)\}$
 $= \lambda x : \neg \mathbf{gap}_{c'}(\mathbf{tall})(x) . \mathbf{pos}_{c'}(\mathbf{tall})(x)$
 where c' is identical to c except that the comparison class in c' is $\{y : \mathbf{pos}_c(\mathbf{tall})(y)\}$

Other degree morphemes could receive a similar treatment. Measure phrases such as *6 feet* could too—it would trigger evaluation with respect to a modified context in which the boundary of the positive extension is drawn at 6 feet.

4.5 Degree functions and comparatives revisited

Klein calls functions that manipulate the extensions of gradable predicates—such as the ones *very* and *6 feet* denote—DEGREE FUNCTIONS. As it turns out, the idea is more generally useful. One application is to the comparative itself. Instead of quantifying over precisifications directly, a comparative could quantify over degree functions:

- (21) a. $\llbracket \text{more} \rrbracket^c$
 $= \lambda f_{(c, et)} \lambda x \lambda y . \exists d \in \mathbf{degree-functions}(c) \left[\begin{array}{l} d(f)(x) \wedge \\ \neg d(f)(y) \end{array} \right]$

$$\begin{aligned}
& \text{b. } \llbracket \textit{more tall} \rrbracket^c \\
& = \lambda x \lambda y . \exists d \in \mathbf{degree-functions}(c) \left[\begin{array}{l} d(\mathbf{tall})(x) \wedge \\ \neg d(\mathbf{tall})(y) \end{array} \right]
\end{aligned}$$

This says that there is a degree function that precisifies gradable predicates in accord with c and would precisify **tall** so that x falls in its positive extension and y doesn't. To put it another way: there is a cut-off (such as 6 feet) that would leave x on the positive side and y on the negative one. This amounts to the same truth conditions as before. So why bother?

One reason is just that it reflects more directly the connection between comparatives, degree morphemes, and measure phrases. Another is that, as it will turn out, this denotation looks very much like a comparative denotation in standard implementations of the degree-based approach to the semantics of gradability, and is therefore important in comparing the two approaches. A third reason is pointed out in [Doetjes et al. \(2011\)](#): many degree functions are ordered with respect to each other (e.g. $\llbracket 5 \textit{ feet} \rrbracket < \llbracket 6 \textit{ feet} \rrbracket$), and this is useful. It draws the two classes of theories even closer together. Degree functions can play many of the roles degrees simpliciter play in the other kind of theory. This other kind of theory is next on the agenda.

5 The degree-based approach

5.1 Degrees

The key element in degree-based theories is, well, DEGREES. What these are precisely can vary from one theory to another, but what they have in common on all of them is that they provide a direct way of representing measurement along a scale. They are all measures of some property. One can be tall to the degree '6 feet', for example, or cold to the degree '-15C'. This way of putting it brings out the other distinguishing element of these theories: they generally treat gradable predicates as having DEGREE ARGUMENTS. Thus *tall*, for example, isn't simply the property tall people have. It's slightly more complex than that. Anyone that is tall is tall to some degree. There is no such thing as being tall without some associated point on a scale. *Tall*, therefore, shouldn't denote a property, but rather a **relation** between individuals and degrees.

An alternative way of construing the same insight links it more closely to resolving the tension between vague language and discrete semantics. One reason fuzzy logic has a certain appeal is that it accords with our feeling that borderline cases satisfy a vague predicate 'less' than clear cases. What we're groping for when we feel this intuition is, perhaps, not the idea that vague predicates fail to yield discrete truth values, but instead that they yield

some abstract measure of the extent to which a gradable property holds. It's not that **tall(Floyd)** yields 1 if Floyd is 7 feet tall and, say, 0.8 if he is 6 feet tall. The very fact that one is forced to pick '0.8' out of thin air—even when we know his precise height—should be alarming. Rather, what we really want to say is that **tall(Floyd)** yields a measure of his tallness: if he's 7 feet tall, '7 feet', and if he's 6 feet tall, '6 feet'. This is not equivalent to the view that gradable predicates denote relations, but it's in the same family. On this version of the theory, articulated in Kennedy (1997), gradable predicates denote MEASURE FUNCTIONS: functions from individuals to degrees.

What's important for current purposes is what these ideas have in common—the notion of degrees, and the idea that a gradable predicate associates an individual with a degree.¹³ There is a great variety of degree-based theories on the market, as this two-pronged introduction begins to attest. Within linguistics, they have proven more popular than inherent-vagueness/supervaluation approaches. One reason may be their general merit, but another is just that they are easier to work with. Research in this tradition is so extensive as to defy easy citation, but includes Seuren (1973), Cresswell (1976), von Stechow (1984), Heim (1985), Bierwisch (1989), Rullmann (1995), Kennedy (1997), Schwarzschild & Wilkinson (2002), Kennedy & McNally (2005), Kennedy (2007) and countless others, many of which will come up throughout the book.

What I'll present here will be a relatively standard exemplar of such a theory, except that it is considerably pared down to avoid presupposing a highly-articulated syntax and to simplify the compositional process. (We will return to those issues in chapter ??.) First, some assumptions about degrees themselves. Intuitively, to measure anything, one needs a SCALE, a kind of an abstract measuring stick. We'll represent a scale as simply a set with certain properties, chief among them that it comes with an ORDERING RELATION, similar to the \leq relation that orders numbers.¹⁴ Degrees are members of such a set. They will not be constructed out of anything else, so they're primitives, atomic types in the model. Not just any set of degrees is a scale, of course. A scale has to be LINEAR, that is, TOTALLY ORDERED: every degree is ordered with respect to every other degree. It is also common to suppose that scales are perfectly gradient rather than granular; that is, the scale has a DENSE ordering relation: for every pair of degrees, however close, there is a degree between them. More formally, degrees are elements of the domain of

¹³These two characteristics go together, but they need not. One could have degrees in the model without using either degree arguments or measure functions, introducing them through some more indirect means. This possibility is explored in Morzycki (2012b).

¹⁴For a bit more on all the order-related terminology here, including more general definitions, consult the glossary.

degrees, D_d ; the variables used for them will be d, d', d'', \dots ; and scales meet the requirements in (22):

- (22) a set of degrees S with the ordering relation \leq is a scale iff $\forall d, d' \in S$:
- a. \leq is total: $d \leq d' \vee d' \leq d$
 - b. \leq is dense: $d \leq d' \rightarrow \exists d'' \in S [d \leq d'' \wedge d'' \leq d']$

Because \leq is a non-strict order, it is also TRANSITIVE, ANTISYMMETRIC, and REFLEXIVE.¹⁵ It has a counterpart $<$ defined in the natural way ($d < d' \stackrel{\text{def}}{=} d \leq d' \wedge d \neq d'$). Neither of the assumptions in (22) is inevitable, and the consequences of eliminating or weakening them are potentially interesting. (For more on (22a) and linearity, see Bale 2011; for more on (22b) and granularity, see Fox & Hackl 2006, Sauerland & Stateva 2007, Nouwen 2008, Abrusán & Spector 2011, van Rooij 2011, and Cummins et al. 2012.)

An important feature of this arrangement is that while all degrees on the same scale can be compared (because they are ordered with respect to each other), degrees can't be compared across scales (because no degrees on different scales are ordered with respect to each other). This means that each scale can be matched to a DIMENSION of measurement: length, temperature, weight, etc. As a consequence, degrees like '6 feet' and '-15C' will remain appropriately distinct and incommensurable.

5.2 Gradable predicates

With these assumptions in place, the denotation of a gradable predicate will be a relation between an individual and a degree (type $\langle d, et \rangle$):¹⁶

¹⁵That is, $\forall d, d', d'' \in S$:

- (i) a. \leq is transitive: $[d \leq d' \wedge d' \leq d''] \rightarrow d \leq d''$
'If one degree is at least as small as a second, and the second at least as small as a third, then the first is at least as small as the third.'
- b. \leq is antisymmetric: $[d \leq d' \wedge d' \leq d] \rightarrow d = d'$
'Two degrees can be at least as small as each other only if they are actually identical.'
- c. \leq is reflexive: $d \leq d$
'Every degree is at least as small as itself.'

The paraphrases are, of course, approximate.

¹⁶A common alternative is to express this relational meaning with the use of a measure function **tallness** (usually written as just **tall**) that maps individuals to their highest degree of height:

- (i) $\llbracket tall \rrbracket = \lambda d \lambda x . \mathbf{tallness}(x) \geq d$

As will emerge from the discussion that follows, this is equivalent to what's intended by the denotation provided in the main text.

$$(23) \quad \llbracket \text{tall} \rrbracket = \lambda d \lambda x . \mathbf{tall}(d)(x)$$

There is an analytical decision to be made here about the relative order of the two arguments: whether the type should be $\langle d, et \rangle$ or $\langle e, dt \rangle$. The choice hinges entirely on what syntactic assumptions one adopts (particularly with respect to whether one assumes a version of the internal subject hypothesis). The type above better accords with a more surface-oriented syntax, for reasons that will become evident.

The first hurdle to get over is how to get from (23) to a denotation for a simple sentence like *Floyd is tall*. In order to get there, it helps to first make a detour that might superficially seem unnecessary into an apparently more complicated structure, the one in (24):

$$(24) \quad \text{Floyd is [six feet tall].}$$

One nice feature of degrees is that we already have a natural denotation for *six feet*, without any need for the more complicated degree functions of the inherent vagueness approach. On the current view, *six feet* can directly denote a degree, type d :

$$(25) \quad \llbracket \text{six feet} \rrbracket = \mathbf{6-feet}$$

From here on, the pieces click into place:

$$(26) \quad \begin{aligned} \text{a. } & \llbracket \text{six feet tall} \rrbracket = \llbracket \text{tall} \rrbracket (\llbracket \text{six feet} \rrbracket) = \lambda x . \mathbf{tall}(\mathbf{6-feet})(x) \\ \text{b. } & \llbracket \text{Floyd is six feet tall} \rrbracket = \llbracket \text{is six feet tall} \rrbracket (\llbracket \text{Floyd} \rrbracket) \\ & = \mathbf{tall}(\mathbf{6-feet})(\mathbf{Floyd}) \end{aligned}$$

It's in this construction and, as we'll see, in the comparative that the degree approach works most straightforwardly.

For the unmodified positive form, more must be said. This might at first seem unintuitive. *Floyd is tall* is, after all, a simpler sentence than *Floyd is six feet tall*, so we might expect a simpler semantics. But this is misleading. Whether the syntax is indeed simpler is not a question that can be resolved at a glance, without investigating it in more detail. Moreover, there is no particular reason in any case to expect that a simpler syntax should necessarily correlate with a simpler semantics. Indeed, the bare positive form is in an important respect manifestly **more** semantically complicated than the measure-phrase form, since only the former is vague.

A better way to think about it is that there are two bits of syntax-semantics that are in complementary distribution (that is, never occur in the same structure): measure phrases and vagueness. When two bits of syntax are in complementary distribution—say, English modal auxiliaries and tense

morphemes—the conclusion to draw is that they compete for the same syntactic position. That’s the conclusion we should draw here, too: vagueness competes for the same syntactic position as the measure phrase. But what does it mean for an abstraction like ‘vagueness’ to compete for a syntactic position? There is only one way to make sense of this: the source of vagueness (of this sort) must be a morpheme in the syntax that is capable of occupying, and therefore competing for, a syntactic position. It has no phonological content, but its semantic content is clearly discernible.

This morpheme is standardly called POS. I’d like to remain relatively neutral about the syntax at this stage in the discussion—despite the syntactic mode of argumentation I just indulged in—but in broad terms, placing it ‘in the same position as’ the measure phrase would yield a structure like (27):

(27) Floyd is [POS tall].

The task this morpheme has to perform is to introduce vagueness. In the inherent vagueness approach, vagueness emerged as a form of context-sensitivity. That will be the case here, too. Again, we will need to index the interpretation function with a context parameter. Instead of retrieving from the context the positive extension of a predicate, on this approach one normally takes a more direct route: the context provides a STANDARD, the smallest degree on a scale consistent with satisfying the predicate—that is, the cut-off point that divides, say, the tall from the non-tall. Thus (27) asserts that Floyd has a degree of height that exceeds the contextually-supplied standard for tallness:¹⁷

(28) $\llbracket \text{Floyd is POS tall} \rrbracket^c = \exists d [d > \text{standard}_c(\text{tall}) \wedge \text{tall}(d)(\text{Floyd})]$

The standard is for the **tall** predicate in context *c* is written here as **standard_c(tall)** (sometimes it is also indicated with a single contextually-supplied variable; ‘norm’ is also occasionally used, following Bierwisch 1989, though that tends to be tied to the idea that the standard retrieved is a ‘normal’ value). Although it’s important to investigate how exactly contexts supply standards, it’s not necessary to have a complete answer to this question in order to make progress. A wide variety of answers are compatible with the framework. All that’s necessary to get off the ground is a placeholder for such answers. In this respect, this isn’t so different from the inherent vagueness theory, which in principle is compatible with various accounts of how a

¹⁷The assumption is often made that it’s sufficient to meet the standard without necessarily exceeding it, in which case the denotation in (28) could be written with \geq in place of $>$ (or even as simply **tall(standard_c(tall))(Floyd)**). These are hard to distinguish empirically.

context determines the cut-off for membership in the positive extension.¹⁸

There is another aspect of (28) that merits attention. The denotation involves existential quantification over degrees. This seems inconsistent with how we normally talk about heights. We usually use definite descriptions such as ‘the height of Floyd’ to talk about heights, not indefinites. So why the existential? There are two ways of addressing this. One pushes back against the objection, and the other embraces it. The first, which is more standard and is the course we will take for now, is that this intuition is at odds with how heights (and therefore degrees) actually behave. Consider a sign next to a roller coaster with the content in (29), along with a horizontal line indicating the required height:

(29) You must be this tall to ride the roller coaster.

The horizontal line is a naturally-occurring counterpart of a degree. Of course, no one would interpret this as admitting only people whose height is precisely the same as the line. Rather, we take anyone who is at least as tall as the line—or taller—to be tall to this degree. We talk in a way that reflects this, too. If the horizontal line is known to be precisely 6 feet off the ground (an odd roller coaster, this), we might even say (30):

(30) It’s obvious Floyd can ride this roller coaster. He’s clearly six feet tall. In fact, he’s at least 6’4”.

So it seems that anyone who is tall to the degree **6-feet** is also tall to every smaller degree. This is sometimes referred to as the **MONOTONICITY** of adjectives.

The principal alternative response to the sense that an individual only has a single height is to encode it into the semantics by having a gradable predicate denote a measure function that returns the **maximal** height of an individual (Kennedy 1997). This makes it possible to avoid existential quantification in the denotation of POS and other degree morphemes, but it requires other additional assumptions to accommodate the fact that gradable predicates are of a type that doesn’t yield truth values (see section ??). Adopting a built-in maximality semantics doesn’t actually require measure function denotations, though (von Stechow 1984, Rullmann 1995; Sharvit & Stateva 2002 have a nice discussion of the issue).

To derive the denotation in (30) compositionally, the contribution of the POS morpheme has to introduce the quantifier and the standard:

¹⁸Indeed, there is nothing to stop us from introducing extension gaps into a degree-based system. One would simply need to introduce two distinct standards, **pos-standard** and **neg-standard**, the cut-offs for membership in the positive and negative extension.

$$(31) \quad \llbracket \text{POS} \rrbracket^c = \lambda G_{(d, et)} \lambda x . \exists d [d > \mathbf{standard}_c(G) \wedge G(d)(x)]$$

The composition process then proceeds as in (32) (I assume *is* is not interpreted):

$$(32) \quad \begin{aligned} \text{a. } \llbracket \text{POS tall} \rrbracket^c &= \llbracket \text{POS} \rrbracket^c (\llbracket \text{tall} \rrbracket^c) \\ &= \lambda x . \exists d [d > \mathbf{standard}_c(\mathbf{tall}) \wedge \mathbf{tall}(d)(x)] \\ \text{b. } \llbracket \text{Floyd is POS tall} \rrbracket^c &= \llbracket \text{POS tall} \rrbracket^c (\llbracket \text{Floyd} \rrbracket^c) \\ &= \exists d [d > \mathbf{standard}_c(\mathbf{tall}) \wedge \mathbf{tall}(d)(\mathbf{Floyd})] \end{aligned}$$

What distinguishes this from the measure-phrase sentence denotation, then—and what introduces vagueness into the picture—is the notion of dependence on contextually-supplied standards.

5.3 *Borderline cases and context-dependence*

There is no single answer in this system to how to handle borderline cases. It isn't tailor-made for that in the way a theory based on extension gaps is. It would be possible to introduce an extension gap into this picture—but equally, it would be possible to accommodate other treatments of borderline cases as well. That's one of the strengths of the approach. The theoretical tools it makes available are versatile, easily adaptable to a wide variety of analytical goals and theoretical and methodological inclinations. The moving parts move smoothly. This certainly helps in syntax-semantics interface questions, as we'll see, but it also helps in adapting the system to broader goals linguists share with philosophers, including acquiring a deeper understanding of the status of borderline cases.

An illustration of this can be found in [Graff \(2000\)](#). She argues that a crucial ingredient in vagueness is the interests of interlocutors in a particular context. Part of what makes us willing to take the steps in a sorites sequence, on this view, is that the steps are not large enough to be salient in the light of those interests. A vague predicate in its positive form therefore requires exceeding a standard to an extent sufficiently large to be salient in the context. This doesn't mean the standard won't still depend on the comparison class as well. Indeed, comparison classes often are provided not by context but by a *for* phrase as in *tall for a basketball player*, so some account of them is necessary on any theory (see also [Kennedy 2007](#), [Bale 2011](#), and [Solt 2011](#)). To reflect these considerations, Graff adopts a POS morpheme that looks roughly like the one in (33).¹⁹ The 'saliently greater than' relation is !>, and

¹⁹I've reframed this to accord with the compositional assumptions in this section. Graff calls her POS morpheme ABS, following [Kennedy \(1997\)](#). This abbreviates 'absolute', another term

norm combines with an adjective (G) and a comparison class property (P) and returns the normal degree associated the comparison class:

$$\begin{aligned}
 (33) \quad & \text{a. } \llbracket \text{POS} \rrbracket^c = \lambda G_{(d,et)} \lambda P_{(e,t)} \lambda x . \exists d [G(x)(d) \wedge d !> \mathbf{norm}(G)(P)] \\
 & \text{b. } \llbracket \text{POS tall} \rrbracket^c = \llbracket \text{POS} \rrbracket^c (\llbracket \text{tall} \rrbracket^c) \\
 & \quad = \lambda P_{(e,t)} \lambda x . \exists d [\mathbf{tall}(d)(x) \wedge d !> \mathbf{norm}(\mathbf{tall})(P)] \\
 & \text{c. } \llbracket \text{POS tall for a basketball player} \rrbracket^c \\
 & \quad = \llbracket \text{POS tall} \rrbracket^c (\llbracket \text{for a basketball player} \rrbracket^c) \\
 & \quad = \llbracket \text{POS tall} \rrbracket^c (\mathbf{basketball-player}) \\
 & \quad = \lambda x . \exists d \left[\mathbf{tall}(d)(x) \wedge \right. \\
 & \quad \quad \left. d !> \mathbf{norm}(\mathbf{tall})(\mathbf{basketball-player}) \right]
 \end{aligned}$$

The result, then, is something that is true of an individual x iff x is tall to a degree that exceeds in a contextually-salient way the normal height for a basketball player. In the absence of the *for* PP spelling out the comparison class, its value is supplied by context.

To be sure, no small part of the theory lies obscured behind the $!>$ symbol, and I have provided only the faintest glimpse of it. But the larger point is that it is easily stated in terms of degrees, and in a way that instantly relates Graff’s subtle philosophical concerns to the grammatical architecture of the expression. Philosophy and syntax have intermingled effortlessly. This is exciting, and is a point in favor of the theoretical framework that brought it about. (See [Stanley 2003](#) and [Kennedy 2007](#) for further discussion relevant to Graff’s approach.)

Inherent vagueness theories, on the other hand, tend to be wedded to a view of vagueness that relies on an extension gap. Graff’s approach has no need—and no use—for one.

The careful reader might observe I have subtly moved the goalposts in this section. The discussion of ‘vague predicates’ in the inherent vagueness theory has turned into a discussion of ‘gradable predicates’ here. This is a reflection of another fact about degree-based accounts. They work beautifully for gradable adjectives, but a theory of other vague predicates—nouns, verbs, even prepositions—doesn’t fall out automatically. Providing such a theory isn’t trivial, but of course that’s precisely what makes it interesting (discussion

for the positive form. One advantage of that term is that it avoids having to describe certain occurrences of the negative member of an antonym pair such as *short* as being a ‘positive negative adjective’. (A disadvantage is that ‘absolute’ is also used in other senses. See the glossary for more terminological griping.) Graff further assumes, with Kennedy, that gradable adjectives denote measure functions, which changes the denotation:

$$(i) \quad \llbracket \text{ABS} \rrbracket = \lambda G_{(e,d)} \lambda P_{(e,t)} \lambda x . G(x) !> \mathbf{norm}(G)(P)$$

can be found in section ?? and in Nakanishi 2004b,a, Morzycki 2005, 2009b, 2012b, Nakanishi 2007, Kennedy & Levin 2008, Rett 2008b, Wellwood et al. 2012, Anderson to appearb).

5.4 *The tautology and contradiction issue*

For the sake of consistency, we should look again at the tautology and contradiction that doomed the fuzzy logic approach, to verify that the degree approach isn't similarly doomed:

- (34) a. Floyd is tall or he isn't tall.
b. Floyd is tall and he isn't tall.

Returning to our simpler original POS morpheme, the denotations would be as in (35):

- (35) a. $\llbracket \text{Floyd is tall or he isn't tall.} \rrbracket^c$
 $= \exists d[\mathbf{tall}(d)(\mathbf{Floyd}) \wedge d > \mathbf{standard}_c(\mathbf{tall})] \vee$
 $\neg \exists d[\mathbf{tall}(d)(\mathbf{Floyd}) \wedge d > \mathbf{standard}_c(\mathbf{tall})]$
 b. $\llbracket \text{Floyd is tall and he isn't tall.} \rrbracket^c$
 $= \exists d[\mathbf{tall}(d)(\mathbf{Floyd}) \wedge d > \mathbf{standard}_c(\mathbf{tall})] \wedge$
 $\neg \exists d[\mathbf{tall}(d)(\mathbf{Floyd}) \wedge d > \mathbf{standard}_c(\mathbf{tall})]$

A quick glance verifies that these have the form $\lceil \phi \vee \neg \phi \rceil$ and $\lceil \phi \wedge \neg \phi \rceil$ respectively—the former necessarily true, the latter necessarily false—so the right result emerges unproblematically.

5.5 *Comparatives*

Comparatives—and other DEGREE CONSTRUCTIONS such as equatives and superlatives—are an area where a degree-based theory excels. One natural way to think about a comparative such *Floyd is taller than Clyde* on this approach is that it requires that there be a degree of tallness that Floyd has and Clyde lacks:

- (36) $\llbracket \text{Floyd is taller than Clyde} \rrbracket^c$
 $= \exists d[\mathbf{tall}(d)(\mathbf{Floyd}) \wedge \neg \mathbf{tall}(d)(\mathbf{Clyde})]$

Compositionally, this can be assembled straightforwardly from the comparative morpheme in (37a) (which I'll again represent as *more*, and again I'll assume *than* is not interpreted):

- (37) a. $\llbracket \text{more} \rrbracket^c = \lambda G_{\langle d, et \rangle} \lambda x \lambda y . \exists d [G(d)(y) \wedge \neg G(d)(x)]$
 b. $\llbracket \text{more tall} \rrbracket^c = \llbracket \text{more} \rrbracket^c (\llbracket \text{tall} \rrbracket^c)$
 $= \lambda x \lambda y . \exists d [\mathbf{tall}(d)(y) \wedge \neg \mathbf{tall}(d)(x)]$
 c. $\llbracket \text{more tall than Clyde} \rrbracket^c = \llbracket \text{more tall} \rrbracket^c (\llbracket \text{than Clyde} \rrbracket^c)$
 $= \lambda y . \exists d [\mathbf{tall}(d)(y) \wedge \neg \mathbf{tall}(d)(\mathbf{Clyde})]$

Because there is no reference to a contextually-provided standard, no entailment is predicted to the positive form. Nothing in (37) tells us anything about whether an individual exceeds that standard for tallness.

As we'll see in chapter ??, it's customary to adopt a more sophisticated syntactic representation than the surface-oriented one adopted in (37), but for the moment this will suffice. It's worth pointing out, though, that one reason degree theories often invoke a more sophisticated syntax is that they're especially good at handling it. In particular, they offer an option that isn't available in principle on the inherent vagueness approach: they can have linguistic expressions denote not just functions that play a degree-like role, but actually degrees themselves. The best an inherent vagueness approach could do would be to have an expression denote a degree function. That's not a bad approximation, of course, but it doesn't have quite the same graceful simplicity.

There are of course other possible treatments of the comparative (as there are for POS). We'll consider alternatives in chapter ?. For now, it's worth just pointing out that many of these use the $>$ relation, which seems especially natural in the context of a degree-based theory. The version here, which Schwarzschild (2008) dubs the 'A-not-A' analysis, nevertheless has much to recommend it and is for that reason widespread. For one thing, given the structure of scales, an equivalent denotation could actually be written that **does** use $>$, just not as simply. Another advantage of this version is that the overt negation explains in an especially direct way why comparative clauses (*than*-clauses) license negative polarity items (NPIs): *than anyone is*, *than he ever has*. As we'll see in section ??, many languages even express comparatives explicitly as coordinate structures with a negated conjunct.

5.6 Degree words

The degree-based theory lends itself very naturally to expressing degree modifiers, a topic to which we will return repeatedly (including in sections 7.2 and 7.4). For the sake of comparison with the inherent vagueness approach, a degree-based denotation for *very* might look like this:

$$(38) \quad \llbracket \text{very} \rrbracket^c = \lambda G_{\langle d, et \rangle} \lambda x . \exists d [G(d)(x) \wedge d \gg_c \mathbf{standard}_c(G)]$$

This is identical to the simple POS denotation, except that it requires exceeding the contextually standard by a large degree, where the context determines what counts as large (this relation is what \gg_c expresses). On this view, a *very* predication is doubly context-sensitive: it relies both on the usual contextually-provided standard and on a contextual definition of what counts as exceeding it by a large amount. There are, however, many other options. In a degree-based theory, one could also express (38) by predicating largeness directly of the difference between a degree and the standard:

$$(39) \quad \llbracket \text{very} \rrbracket^c = \lambda G_{(d, et)} \lambda x . \exists d [G(d)(x) \wedge \mathbf{large}_c(d - \mathbf{standard}_c(G))]$$

Yet another alternative is to simply adopt Klein's *very* into this framework, which can also be done straightforwardly (Kennedy & McNally 2005).

5.7 Varieties of degrees

As presented here, degrees are atomic types, simply points on a scale abstractly representing measurement. This is not the only option. We'll encounter a various alternatives over the course of the book, but it makes sense to mention a few of them now to convey a sense of the available options.

First, one could represent degrees not as points rather as intervals, sets of points portions of a scale (Kennedy 1997, Schwarzschild & Wilkinson 2002). This arguably makes some of the system simpler, and may have welcome consequences for scope ambiguities, measure phrase licensing, and capturing the distinction between antonymous adjectives (e.g. *short* vs. *tall*; for scope issues, see section ??, and for the others, section 7.1).

One could also adopt an earlier idea, due to Cresswell (1976), that constructs degrees out of equivalence classes of individuals. An equivalence class is a set of individuals that have the same measure along some dimension: height, weight, size, pleasantness, etc. (More formally, an equivalence class is any subset whose members stand in an equivalence relation to each other, where an equivalence relation—like 'has the same height as'—is like a partial order except that instead of being antisymmetric, it is symmetric.) This has the advantage of metaphysical parsimony. It would mean there is one fewer atomic type in the model. This approach yields a less flexible notion of degrees, though. It has trouble with the meaning of measure phrases in differential comparatives such as *Floyd is two feet taller than Clyde*, where *two feet* couldn't plausibly denote everything that measures two feet and it's not clear how to achieve the effect of adding or subtracting degrees. In light of this, it wouldn't be unreasonable to doubt whether such a theory really counts as a degree-based theory.

Some other possibilities:

- model at least some degrees as numerically (using real or rational numbers depending on the author; Hellan 1981, Bale 2006, 2008)
- model degrees as concrete property manifestations ('tropes') of the sort that e.g. *the height of Floyd* might refer to (Moltmann 2009, 2007)
- model degrees as kinds of states (Anderson & Morzycki 2012)
- construct degrees out of several more basic elements, such as a property, a measure, and a measured object (Grosu & Landman 1998)
- recognize more than one type of degree (Bale 2006, 2008; see section ??)

For most purposes, the standard approach sketched in the preceding sections is easiest to work with.

6 Degree or not degree? That is the question

Now that both the inherent vagueness and degree-based approaches are on the table, we can consider them in relation to each other.

The differences between the approaches at first glance seem profound. This is at least partly misleading. The most important difference is probably that one theory treats degrees as objects in the model and makes use of degree arguments. But of course, to say that degrees are 'objects in the model' is not to say much, given that they don't need to be primitives (i.e., atomic types). If a theory in which degrees are constructed out of something else counts as a degree theory, well, then it has that in common with an inherent vagueness theory that has degree functions, which are also 'in the model' but not atomic types. Of course, the types involved in degree functions are more complicated, but perhaps that's not particularly important.

A point of clear similarity is their treatment of comparatives, at least as I have presented them here (van Rooij 2008, Doetjes et al. 2011, and Nouwen et al. 2011 make a similar point):

- (40) Floyd is taller than Clyde.
- inherent vagueness theory:*
 $\exists d \in \mathbf{degree-functions}(c)[d(\mathbf{tall})(\mathbf{Floyd}) \wedge \neg d(\mathbf{tall})(\mathbf{Clyde})]$
 - degree-based theory:*
 $\exists d[\mathbf{tall}(d)(\mathbf{Floyd}) \wedge \neg \mathbf{tall}(d)(\mathbf{Clyde})]$

This similarity is precarious, and it could melt away with only minor changes. Nevertheless, many changes in one theory might find analogues in the other, since both are manipulating something like degrees.

One deep difference between the two classes of approaches is that an extension gap theory is tightly bound to a particular view of vagueness. One could certainly enrich it in various ways, but it would lose a major component of its character if the extension gap element were gone. What's especially troubling about this is that extension gaps have significant shortcomings as a theory of vagueness. They certainly reflect the existence of borderline cases, but what about the boundary between borderline and clear cases? The theory suggests that it should be completely sharp, but that's not consistent with our intuitions. There are certainly some borderline cases that are clearly borderline cases, but there are also ones that are borderline cases of borderline cases. This phenomenon, called HIGHER ORDER VAGUENESS, strikes at the heart of the theory. If vagueness is simply due to extension gaps, what accounts for vagueness about the extension gaps themselves?

Additional difficulties are pointed out in [Kennedy \(1997\)](#). One of them involves incommensurability, the ill-formedness of comparatives (and related constructions) formed from adjectives that measure along different dimensions:

- (41) #My copy of *The Brothers Karamazov* is heavier than my copy of *The Idiot* is old. ([Kennedy 1997](#))

It's not that inherent-vagueness theories leave no room for explanations of such effects. The problem with these examples, one might say, is conceptual rather than semantic. Perhaps one just can't make sense of a comparison between weight and age? If that were so, however, we would expect (42) to be just as bad:

- (42) My copy of *The Brothers Karamazov* is higher on a scale of heaviness than my copy of *The Idiot* is on a scale of age. ([Kennedy 1997](#))

Yet this sentence is fine—or rather, it's odd in precisely the way a conceptual oddness might feel. It seems strange that anyone would want to make such a comparison, but there is no sense of semantic anomaly. He notes further problems having to do with several varieties of comparative. The most important property of inherent vagueness theories that gives rise to these problems is that they don't offer a sufficiently articulated notion of scales. To be sure, they involve orderings among individuals, orderings that are present in the model itself. But on these theories the comparative (ultimately) involves quantification over precisifications rather than over degrees on a particular scale, so arbitrary cross-scale comparisons are expected to licit. Moreover, operations that are easily defined in a degree theory—such as measuring the difference between degrees in e.g. differential comparatives like *two feet taller*—are problematic.

On the other hand, inherent vagueness theories have at least two apparent advantages over degree theories. First, they take the positive form of the adjective to be basic, and define the comparative in terms of it. A degree theory, arguably, does precisely the opposite, because it assigns a semantics to a positive adjective that involves an ordering relation: a positive adjective has a meaning of the form ‘more *G* than the standard for *G*’. Yet across languages, the positive form is the less syntactically complicated one (or in any case, that’s what the syntax superficially suggests). This is potentially a deep problem, and an oft-mentioned one. In principle, one can imagine a vaguely functionalist response that goes like this: In any language, the more often-used form is likely to be the one that involves the most phonological and syntactic reduction, and the less often-used one will be the one with overt bells and whistles. It may simply be that positive adjectives are more common than comparatives, across languages, and apparently structurally simpler for that reason alone. But that doesn’t necessarily tell us anything about the relative complexity of the semantics—for insight into that, we must ask about truth conditions, not count overt morphemes.

Second, inherent vagueness theories provide a better understanding of the ubiquity of vagueness. It’s not just adjectives that are vague, after all. A degree-based theory weds vagueness to degree arguments, and therefore in turn to gradability. It’s certainly true that vagueness and gradability are closely related, but they’re not indistinguishable. *Heap*, for example, isn’t directly gradable.²⁰ So, on a degree-based theory, what is to be said about it? Do we give it a degree argument, and thereby—in the face of the grammatical evidence—a gradable semantics? That seems unappealing, particularly when it has to be extended to other syntactic categories as well. If the answer is no, then an independent theory of the vagueness of these expressions is required. Whatever that independent theory is, it would likely result in an account of adjectival vagueness too, which would mean adjectival vagueness would be explained twice over. The generative linguist’s instinctive drive for simplicity recoils at this possibility. On an inherent vagueness theory, vagueness is treated as a single unified phenomenon, and these issues don’t arise.

This too is a deep problem. Again, though, one could argue the other way: we know adjectives are the primary gradable category. A theory should reflect that, as introducing degree arguments does. Any theory that levels the distinction between gradable adjectives and non-gradable vague predicates in other categories fails to explain why adjectives are so good for grading. And here again a reply is available: what’s special about adjectives that makes them especially gradable is a subtle fact about their lexical semantics, not a

²⁰It can enter into constructions such as *more of a heap*, of course.

crude fact about their type: they have meanings that just lend themselves, conceptually, to gradability.

This dialogue could continue, and no doubt should (elsewhere). But before leaving it, it's worth pointing out an intermediate position. Perhaps **both** approaches are right. Although this sounds superficially like a mealy-mouthed compromise, one shouldn't reject it out of hand. Running the theories in parallel would decouple general vagueness and gradability. That might be what's empirically necessary. Caution is warranted, of course. Combining competing theories can yield the union of their flaws and the intersection of their virtues. Even so, there might be a way to, as [Doetjes et al. \(2011\)](#) suggest, integrate them insightfully. One might, for example, eliminate the extension gaps themselves, but maintain the idea that vague predicates lack degree arguments and have their extensions fixed by context—and perhaps even that gradability can be understood in terms of degree functions and quantification over precisifications.

7 Scales and the lexical semantics of adjectives

7.1 Antonyms

Chapter ?? focused on non-scalar issues in the lexical semantics of adjectives. We now have the tools in place to handle the scalar ones too. Among the more obvious of these is the relation between adjectives and their antonyms:

- (43) tall ↔ short
wide ↔ narrow
old ↔ young
fast ↔ slow
hot ↔ cold

Many adjectives are like this. Unsurprisingly, the marked member of each pair is called a NEGATIVE ADJECTIVE. This is a terrible term. It makes the other member of the pair, inevitably, a POSITIVE adjective. This term is already used to designate the morphologically unmarked form of the adjective too, so some negative adjectives occur in the positive form. This might suggest that we should reserve 'positive' for this sense and refer to the morphologically unmarked form as just 'unmarked', except that the term 'unmarked' is taken, too—and for precisely this purpose. *Tall* and *short* are the 'unmarked' and 'marked' members of an antonym pair. There's nothing to be done but to press on.

The first question to ask about this distinction is how one knows which member of the pair is the negative one. Intuitions about being 'positive' are

not an adequate guide. There are some clear diagnostics, though (Seuren 1978, Bierwisch 1989, Kennedy 2001, Rett 2008a,b, Sassoon 2010a). Negative adjectives never accept measure phrases (setting aside comparatives):

- (44) a. six feet $\left\{ \begin{array}{l} \text{tall} \\ \# \text{short} \end{array} \right\}$
 b. six feet $\left\{ \begin{array}{l} \text{wide} \\ \# \text{narrow} \end{array} \right\}$
 c. six years $\left\{ \begin{array}{l} \text{old} \\ \# \text{young} \end{array} \right\}$

Of course, many positive adjectives don't accept measure phrases either: *#80 mph fast*, *#-15C cold*. Negative adjectives are also dispreferred with factor phrases like *twice* in the equative:

- (45) twice as $\left\{ \begin{array}{l} \text{tall} \\ ? \text{short} \\ \text{wide} \\ ? \text{narrow} \\ \text{old} \\ ? \text{young} \end{array} \right\}$

Negative adjectives don't occur in nominalizations that name the dimension along which they measure:

- (46) The $\left\{ \begin{array}{l} \text{length} \\ \# \text{shortness} \\ \text{width} \\ \# \text{narrowness} \end{array} \right\}$ of the coffee table is 4 feet.

In *wh*-questions, negative adjectives give rise to a presupposition:

- (47) a. How $\left\{ \begin{array}{l} \text{tall} \\ \text{short} \end{array} \right\}$ are you?
 b. How $\left\{ \begin{array}{l} \text{wide} \\ \text{narrow} \end{array} \right\}$ is this coffee table?

Short in (47a) gives rise to the presupposition that you're short, and *narrow* in (47b) to the presupposition that the coffee table is narrow. No analogous presupposition arises for the positive form. A similar effect happens in the equative:

- (48) a. Floyd is as $\left\{ \begin{array}{l} \text{tall} \\ \text{short} \end{array} \right\}$ as Clyde.
 b. The coffee table is as $\left\{ \begin{array}{l} \text{wide} \\ \text{narrow} \end{array} \right\}$ as the couch.

Short in (48a) gives rise to the entailment that Floyd and Clyde are short, and *narrow* in (48b) to the entailment that the coffee table and couch are narrow.

A more subtle but notable difference is that negative adjectives in the comparative can give rise to ambiguities involving modals, sometimes called the (Seuren-)Rullmann ambiguity (Seuren 1978, Rullmann 1995, Heim 2006, Rett 2008b, Beck 2013):

- (49) The helicopter was flying lower than a plane can fly. (Rullmann 1995)
 a. The helicopter was flying lower than the the **lowest** level a plane can fly.
 b. The helicopter was flying lower than the the **highest** level a plane can fly.

The corresponding positive form (i.e., with *higher*) is unambiguous.

These all serve both as diagnostics for the negative member of a pair, and as data to be explained. One additional fact needs to be added to this mix. Antonym pairs can give rise to CROSS-POLAR ANOMALY (so dubbed by Kennedy 1997, 2001) in comparatives:

- (50) a. ??Floyd is shorter than Clyde is tall.
 b. ??This table is wider than that one is narrow.
 c. ??Your nephew is younger than your grandmother is old.

These reflect that the adjective in the matrix and comparative clause must either both be positive or both be negative. Buring (2007a) observes that sentences with essentially the same meaning are improved when one of the adjectives isn't the polar antonym:

- (51) a. The ladder is shorter than the house is high. (Buring 2007a)
 b. My yacht is shorter than yours is wide.

This also demonstrates that the problem in (50) can't be due to a constraint on forming comparatives with non-identical adjectives.

Kennedy (2001) proposed an account of these facts based on a particular ontology of degrees. The idea is that there are two sorts of degrees: positive and negative, both of which are intervals. Positive adjectives measure in

positive degrees, and negative adjectives measure in negative degrees. The two sorts are related, obviously. They use the same sets of points, but they are on different scales because they have different orderings, one the mirror-image of the other. Given this way of thinking, then, ‘Floyd’s tallness’ and ‘Floyd’s shortness’ are distinct degrees. ‘Floyd’s tallness’ is what one might expect: if he’s six feet tall, it’s a positive degree that extends upwards from (just above) 0 to 6 feet. ‘Floyd’s shortness’, however, is a little surprising. It extends to 6 feet, but, because it’s negative, it gets there from the opposite direction—from above—by extending **downwards**. This of course means it can’t start at 0. The scale has no opposite end, however: there is no maximum height in principle. So the degree of his shortness extends from infinity down to 6 feet:

- (52) a. Floyd’s tallness: (0, 6ft]
 b. Floyd’s shortness: (∞ , 6ft]

Two results follow naturally from this very intuitive set-up. First, cross-polar anomaly is explained straightforwardly: no ordering is defined between positive and negative degrees. Attempting to compare Floyd’s tallness to Clyde’s shortness would require precisely such a comparison, as (53) reflects:

- (53) a. #Floyd is taller than Clyde is short.
 b. $\exists d[\mathbf{tall}(d)(\mathbf{Floyd}) > \mathbf{short}(d)(\mathbf{Clyde})]$

The result of this is, of course, undefined. A rough intuitive approximation of the idea is simply that there’s no way to compare how tall people are to how tall they aren’t. Second, the measure phrase facts also fall into place. The way this emerges is that measure phrases denote positive rather than negative degrees. It would be odd if it were otherwise: measure phrases have to start measuring relative to a fixed point, so they must denote intervals that extend from an origin point on a scale. A negative interval of this sort wouldn’t have one.

Heim (2006) (see also Büring 2007b, Heim 2008) takes a different approach, building on Rullmann (1995). She takes as her point of departure the Rullmann ambiguity in (49), and winds up with a syntactic rather than ontological solution. It is based on two intuitions. First, we do want something that resembles negation associated with negative adjectives. In many cases, negative adjectives have overtly negative morphology (e.g. *impure*, *unmanageable*, *implausible*). Second, as Rullmann noticed, precisely the same ambiguity emerges when *lower* is replaced by *less high*:

- (54) The helicopter was flying less high than a plane can fly.
(Rullmann 1995)
- a. The helicopter was flying lower than the the **lowest** level a plane can fly.
 - b. The helicopter was flying lower than the the **highest** level a plane can fly.

This suggests that *lower* spells out the same structure as *less high* does. But these forms are both in the comparative. Obviously, not all negative adjectives are comparative. So if we are to generalize this decomposition, we need to find a way to factor out the comparative morphology. A way to do that is revealed by paradigms like those in (55):

- (55) a. He $\left\{ \begin{array}{l} \text{knows} \\ \text{expects} \end{array} \right\} \left\{ \begin{array}{l} \text{little} \\ \text{less} \\ \text{the least} \end{array} \right\}$.
- b. We have $\left\{ \begin{array}{l} \text{little} \\ \text{less} \\ \text{the least} \end{array} \right\}$ water.

In this way, *little* is a counterpart of *much*:

- (56) a. He $\left\{ \begin{array}{l} \text{knows} \\ \text{expects} \end{array} \right\} \left\{ \begin{array}{l} \text{much} \\ \text{more} \\ \text{the most} \end{array} \right\}$.
- b. We have $\left\{ \begin{array}{l} \text{much} \\ \text{more} \\ \text{the most} \end{array} \right\}$ water.

This shows that *less* is simply the comparative form of *little*: *-er little* (Bresnan 1973). This in turn means *less high* is really *-er little high*, as in (57a). And if *lower* spells out the same structure as *less high*, it too must be *-er little high* underlyingly, as in (57b):²¹

- (57) a. *[-er little] high* \Rightarrow *less high*
 b. *-er [little high]* \Rightarrow *-er low* \Rightarrow *lower*

So we've arrived at a way to factor out the comparative from *lower*: *low* alone is *little high*. It can't actually be pronounced that way in natural-sounding English, but this may be a morphological quirk of the language. German,

²¹I have switched from representing the comparative morpheme as *more* to *-er* since, on this view, *more* could be *-er much*.

Heim says, permits it. This how [Rullmann \(1995\)](#) explains why *less high* and *lower* both give rise to the Rullmann ambiguity. Heim considers this approach but ultimately rejects it, but it is embraced and further developed in [Büring \(2007a,b\)](#) (see also [Heim 2008](#)).

This decomposition is of course just a start. The actual scope-taking machinery necessary in to account for the ambiguity is complex and presupposes syntactic and semantic assumptions we won't have in place until the following chapter (see [Heim 2006, 2008](#) and [Büring 2007a,b](#)). Nevertheless, it's possible to sketch an analysis that connects to the basic insight behind [Heim \(2006\)](#), which is this: *little* expresses a mode of negation specialized for degrees and gradable predicates. In our terms, it might look like (58a), meaning that *low* (underlyingly *little high*) will have the denotation in (58b) (we will now depart significantly from Heim):

$$(58) \quad \begin{aligned} \text{a. } \llbracket \textit{little} \rrbracket &= \lambda G_{\langle d, et \rangle} \lambda d \lambda x . \neg G(d)(x) \\ \text{b. } \llbracket \textit{little high} \rrbracket &= \llbracket \textit{little} \rrbracket (\llbracket \textit{high} \rrbracket) \\ &= \lambda d \lambda x . \neg \llbracket \textit{high} \rrbracket (d)(x) \\ &= \lambda d \lambda x . \neg \mathbf{high}(d)(x) \end{aligned}$$

To get to a sentence denotation, of course, we must go via POS. Our current POS denotation won't suffice, however, since it would predict that an individual x is POS *little high* iff there's a degree above the standard to which x isn't high, as in (59) (I'll omit context super/subscripts from now on):

$$(59) \quad \begin{aligned} \text{a. } \llbracket \text{POS} \rrbracket &= \lambda G_{\langle d, et \rangle} \lambda x . \exists d [G(d)(x) \wedge d > \mathbf{standard}(G)] \\ \text{b. } \llbracket \text{POS } \textit{little high} \rrbracket &= \llbracket \text{POS} \rrbracket (\llbracket \textit{little high} \rrbracket) \\ &= \lambda x . \exists d \left[\llbracket \textit{little high} \rrbracket (d)(x) \wedge \right. \\ &\quad \left. d > \mathbf{standard}(\llbracket \textit{little high} \rrbracket) \right] \\ &= \lambda x . \exists d \left[\neg \mathbf{high}(d)(x) \wedge \right. \\ &\quad \left. d > \mathbf{standard}(\lambda d \lambda x . \neg \mathbf{high}(d)(x)) \right] \end{aligned}$$

This is far too weak. Unless x is high to every degree, there will always be a degree above a standard to which x isn't high.

Heim instead adopts an alternative denotation advanced by [von Stechow \(2005\)](#) (see also [Beck 2011](#)). Von Stechow suggests that positive and negative adjectives both use the same scale, with two cut-offs. For example, being low might require being below 12 feet, and being high might require being at at least 100 feet. Von Stechow calls this middle ground between the antonyms the 'delineation interval', which I'll represent with a predicate

middle-ground.²² His POS, modified significantly to match our current assumptions, requires that an individual satisfy a gradable predicate to all the degrees in the middle ground:

- (60) a. $\llbracket \text{POS} \rrbracket = \lambda G_{\langle d, et \rangle} \lambda x . \forall d \in \mathbf{middle-ground}[G(d)(x)]$
 b. $\llbracket \text{The helicopter is POS high.} \rrbracket$
 $= \forall d \in \mathbf{middle-ground}[\mathbf{high}(d)(\mathbf{the-helicopter})]$

Thus the helicopter is high iff it is high to every degree from the *low* cut-off to the *high* cut-off—so it must be at least as high as the *high* cut-off. Combining this with *little* yields a semantics for *low*:

- (61) a. $\llbracket \text{little high} \rrbracket = \lambda d \lambda x . \neg \mathbf{high}(d)(x)$
 b. $\llbracket \text{The helicopter is POS little high.} \rrbracket$
 $= \forall d \in \mathbf{middle-ground}[\neg \mathbf{high}(d)(\mathbf{the-helicopter})]$

Thus the helicopter is low iff it fails to be high to any degree from the *low* cut-off to the *high* cut-off—so it must be less high than the *low* cut-off.²³

In the comparative, the pieces fit together elegantly:

- (62) a. $\llbracket \text{-er} \rrbracket = \lambda x \lambda y . \exists d [\mathbf{high}(d)(y) \wedge \neg \mathbf{high}(d)(x)]$
 b. $\llbracket \text{-er little high} \rrbracket = \lambda x \lambda y . \exists d \left[\begin{array}{l} \llbracket \text{little high} \rrbracket (d)(y) \wedge \\ \neg \llbracket \text{little high} \rrbracket (d)(x) \end{array} \right]$
 $= \lambda x \lambda y . \exists d [\neg \mathbf{high}(d)(y) \wedge \neg \neg \mathbf{high}(d)(x)]$
 $= \lambda x \lambda y . \exists d [\neg \mathbf{high}(d)(y) \wedge \mathbf{high}(d)(x)]$
 c. $\llbracket \text{The helicopter is -er little high than the plane} \rrbracket$
 $= \exists d [\neg \mathbf{high}(d)(\mathbf{the-helicopter}) \wedge \mathbf{high}(d)(\mathbf{the-plane})]$

Thus the helicopter is lower than the plane iff there's a degree to which the plane is high to which the helicopter isn't. This is the right result. It's not a complete theory of the Rullmann ambiguity, but by enriching this with more sophisticated compositional assumptions, one can reassemble these basic ingredients in multiple ways that do provide such a theory (see Rullmann 1995, Heim 2006, 2008, Buring 2007a,b).

²²I'll leave off any indication of which scale is at issue, though this could be done by simply providing the gradable predicate as an argument.

The delineation interval is a little like an extension gap, but not quite. First, it's an interval on a degree scale, not a set of individuals. Second, it demarcates the area between e.g. *low* and *high*, not between *not high* and *high*.

²³This all predicts an asymmetry between the two cut-offs: being precisely at the *low* cut-off doesn't render you *low*, but being precisely at the *high* cut-off does render you high.

7.2 Open and closed scales

Over roughly the past decade, it has emerged that another semantic distinction among adjectives—indeed, a range of predicates—is no less important than antonymy. It can be glimpsed in the contrasts reflected in (63) and (64):

- (63) a. The glass is $\left\{ \begin{array}{l} \text{half} \\ \text{mostly} \end{array} \right\}$ full.
b. Her eyes were $\left\{ \begin{array}{l} \text{half} \\ \text{mostly} \end{array} \right\}$ closed.
c. These images are $\left\{ \begin{array}{l} \text{half} \\ \text{mostly} \end{array} \right\}$ invisible.
- (64) a. ??A 15-year-old horse is $\left\{ \begin{array}{l} \text{half} \\ \text{mostly} \end{array} \right\}$ old.
b. ??That car was $\left\{ \begin{array}{l} \text{half} \\ \text{mostly} \end{array} \right\}$ expensive.
c. ??Clyde seemed $\left\{ \begin{array}{l} \text{half} \\ \text{mostly} \end{array} \right\}$ tall.

The PROPORTIONAL MODIFIERS *half* and *mostly* turn out to be just the tip of the iceberg, a reflection of a distinction with broader consequences. But what is this distinction, precisely? How should it be represented formally?

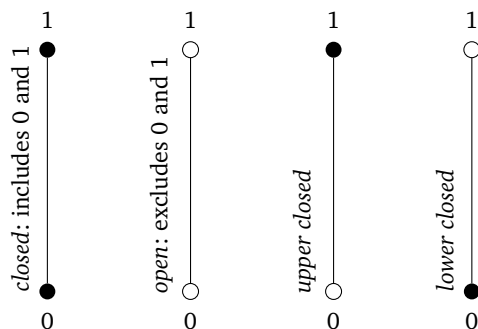
Kennedy & McNally (2005) and Rotstein & Winter (2004) provide an answer: these adjectives differ in the structure of their scales. (I'll frame the discussion along the lines of the former.) There are many aspects of how scales are organized that one might describe as 'scale structure', but the one that's relevant here has to do with what happens at the ends of a scale. There are four options, which can be best appreciated by thinking degrees on a scale in terms of real numbers between 0 and 1. One option is for a scale to include 0 and 1 in addition to the numbers between them. This would be a CLOSED SCALE, so called because it is a closed interval: one that includes minimal and maximal values. The natural alternative is for a scale to exclude 0 and 1, including only the real numbers between them. This is an OPEN SCALE. It doesn't include a minimal or maximal value. It **approaches** 0 and 1 at its extremes, but never reaches them—there is no smallest non-zero between 0 and 1, and no largest non-one number either. There are, of course, two other possibilities—a scale could include 1 and leave out 0, or vice versa. This can be stated set-theoretically:

(65) SCALE TYPES

- a. *closed*: $\{d : 0 \leq d \leq 1\}$
- b. *open*: $\{d : 0 < d < 1\}$
- c. *upper closed*: $\{d : 0 < d \leq 1\}$
- d. *lower closed*: $\{d : 0 \leq d < 1\}$

Here is a visual representation:

(66) SCALE TYPES



Because these are intervals, a standard notation for intervals can be used. The closed interval is $[0, 1]$, the open one $(0, 1)$, the upper closed $(0, 1]$, and the lower closed $[0, 1)$.

This is purely a formal distinction, which might well have turned out to be linguistically irrelevant. But it isn't. It provides a way of representing scale boundedness, the intuition that scales can vary with respect to whether they have a highest or lowest degree. That's the idea behind Kennedy & McNally's account of proportional modifiers. Proportions are about bounded quantities. If I ask you how much coffee you'd like, you can't reasonably reply 'half'. Analogously, the degree modifier *half* also needs a bounded quantity, in this case, a bounded—that is, closed—scale.

This can be represented in the semantics straightforwardly. The intuition behind *half* is that it locates a degree whose distance from the bottom of a scale (its minimal degree, written as a function **min** that applies to scales) is the same as the distance from the top (its maximal degree, **max**).²⁴ To say this more formally, we'll need a **scale** function that applies to a gradable predicate and returns its scale, and a degree difference operation indicated

²⁴The **min** and **max** functions will reoccur elsewhere in the book. Their definitions (S is a set of degrees or a scale construed as such):

- (i) a. $\mathbf{max}(S) \stackrel{\text{def}}{=} } \iota d [d \in S \wedge \forall d' \in S [d' \leq d]]$
'the unique degree in S such that every degree in S is smaller than (or identical to) it'

with $-$. *Half*, then, will be as in (67a), and precisely the same tools give us *mostly* as well:

$$(67) \quad \begin{array}{l} \text{a. } \llbracket \textit{half} \rrbracket = \lambda G_{\langle d, et \rangle} \lambda x . \exists d \left[\begin{array}{l} G(d)(x) \wedge \\ \mathbf{max}(\mathbf{scale}(G)) - d = \\ d - \mathbf{min}(\mathbf{scale}(G)) \end{array} \right] \\ \\ \text{b. } \llbracket \textit{mostly} \rrbracket = \lambda G_{\langle d, et \rangle} \lambda x . \exists d \left[\begin{array}{l} G(d)(x) \wedge \\ \mathbf{max}(\mathbf{scale}(G)) - d > \\ d - \mathbf{min}(\mathbf{scale}(G)) \end{array} \right] \end{array}$$

These are like POS in that they are degree words and they saturate the degree argument. Taking one additional step:

$$(68) \quad \llbracket \textit{half} \rrbracket (\llbracket \textit{full} \rrbracket) = \lambda x . \exists d \left[\begin{array}{l} \mathbf{full}(d)(x) \wedge \\ \mathbf{max}(\mathbf{scale}(\mathbf{full})) - d = \\ d - \mathbf{min}(\mathbf{scale}(\mathbf{full})) \end{array} \right]$$

So something is half full if it is full to a degree that is the same distance from the minimum and maximum of the scale.

These denotations also explain what goes wrong for adjectives incompatible with proportional modifiers—that is, for adjectives with open or partly open scales. In a case like *half old*, the maximality and minimality operators will apply to the scale of age. But (at least) the maximality operator simply isn't defined for the scale of age because it doesn't have a maximum. The sentence, therefore, comes out undefined.

Similar reasoning can accommodate modifiers that are sensitive to only one end of the scale. The MAXIMALITY MODIFIERS *fully* and *completely* are of this class:

$$(69) \quad \text{CLOSED SCALE}$$

$$\begin{array}{l} \text{a. The flower was fully } \left\{ \begin{array}{l} \text{open} \\ \text{closed} \end{array} \right\}. \\ \\ \text{b. The monkey was fully } \left\{ \begin{array}{l} \text{visible} \\ \text{invisible} \end{array} \right\}. \end{array}$$

b. $\mathbf{min}(S) \stackrel{\text{def}}{=} \iota d [d \in S \wedge \forall d' \in S [d \leq d']]$
 'the unique degree in S that smaller than (or identical to) any degree in S '

(70) OPEN SCALE

- a. ??Floyd is fully $\left\{ \begin{array}{l} \text{tall} \\ \text{short} \end{array} \right\}$.
- b. ??This table is fully $\left\{ \begin{array}{l} \text{wide} \\ \text{narrow} \end{array} \right\}$.

(71) UPPER CLOSED SCALE VS. LOWER CLOSED SCALE

- a. We are fully $\left\{ \begin{array}{l} \text{certain} \\ \text{??uncertain} \end{array} \right\}$.
- b. The treatment is fully $\left\{ \begin{array}{l} \text{safe} \\ \text{??unsafe} \end{array} \right\}$.

The examples in (71) require special attention. The first adjective in each pair is upper-closed (lower open), and its antonym is lower-closed (upper open). This all reveals that *fully* has a semantics that requires reference to the maximum on a scale, but not to a minimum:

(72) $\llbracket \text{fully} \rrbracket = \lambda G_{\langle d, et \rangle} \lambda x . G(\mathbf{max}(\mathbf{scale}(G)))(x)$

Consequently, *fully* will be incompatible with any scale for which a maximum degree isn't defined, but it will be indifferent to the presence of a minimum degree. Sometimes *slightly* is suggested as an example of a modifier that requires lower-closed scales, but the judgments it evokes are less clear and Kennedy & McNally don't mention it.

There is a more important insight to be gleaned from (72), however. The distribution of degree modifiers shows that the scales of polar antonyms are identical except in direction of the ordering—metaphorically, in which end is up. The maximum of one scale is the minimum of the other, and vice versa. If an adjective has a maximum, its antonym will always have a minimum. This is an important insight relevant to a general theory of antonymy, and thus has consequences far beyond the distribution of the degree modifiers.

The semantics of positive forms is one area where these differences turn out to be crucial:

(73) UPPER CLOSED SCALE VS. LOWER CLOSED SCALE

- a. The rod is $\left\{ \begin{array}{l} \text{straight} \\ \text{bent} \end{array} \right\}$.
- b. The soap is $\left\{ \begin{array}{l} \text{pure} \\ \text{impure} \end{array} \right\}$.
- c. The child is $\left\{ \begin{array}{l} \text{healthy} \\ \text{sick} \end{array} \right\}$.

All of these adjectives have partially closed scales (as combining them with e.g. *fully* would reveal). In each of these cases—and more generally—the standard associated with the scale always corresponds to the closed end. For example, *straight* is upper closed and lower opened, and *bent* is therefore the opposite. Across contexts, the standard for *straight* is set at the maximum on the scale: something is straight iff it's fully straight. *Bent* is a mirror image. Across contexts, the standard for *bent* will be set at the minimum on the scale: something is bent iff it has any amount of bend at all.²⁵ For fully closed scales, there is a complication: there are two natural endpoints. In these cases, the adjective must simply resolve the matter lexically. Open-scale adjectives pose the opposite problem: not a surfeit of endpoints but too few. *Tall* and (therefore) *short* are open scale, so their scales include no natural boundary one might use as a standard. Without reliance on context, there is no way to determine conclusively where the standard lies. And so, in these cases, that's precisely what we do—rely on context, giving rise to vagueness.

Because of this fundamental difference, Kennedy & McNally dub adjectives with at least partly closed scales ABSOLUTE ADJECTIVES because their standard is fixed at the closed end of the scale. Open-scale adjectives like *tall* have no closed end, so their standard is context-dependent. They dub these RELATIVE ADJECTIVES.²⁶ Of course, absolute adjectives can be subdivided further, into those with minimum standards and those with maximum ones. Older terms for these, which are still very much in use, are PARTIAL and TOTAL adjectives (Yoon 1996, Rotstein & Winter 2004). Fully closed-scale adjectives are a bit less tidy: they are absolute adjectives, and their standard is always at one end, but which end needs to be stipulated in the lexicon.

To account for this effect, the POS morpheme must be changed. As it stands, it requires exceeding a contextually-provided standard. This is doubly problematic. First, it suggests all adjectives should be context-dependent, not just relative adjectives. Second, for maximum standard adjectives, it would impose an impossible to meet requirement: if the standard is at the top of a scale, it's impossible to exceed it. That could be addressed by changing the ordering relation from $>$ to \geq , so that it's only necessary to meet the standard rather than exceed it. But this leaps out of the frying pan and into the fire: for minimum-standard adjectives, any degree on the scale meets or exceeds the standard, so any positive-form predication involving a minimum-standard adjective would be true. Kennedy & McNally propose working around this

²⁵It's possible to imagine contexts in which one might say something is straight even if it has a tiny, pragmatically irrelevant amount of bend. Kennedy & McNally argue convincingly that this involves imprecision, not vagueness (see section 2.3).

²⁶Neither term is ideal because both have other uses. 'Absolute' is sometimes used to refer to the positive form of an adjective (Kennedy 1997), and 'relative' is sometimes used as a synonym for 'gradable' or for 'subjective' (Bartsch & Vennemann 1973, Siegel 1976).

by encoding the degree ordering relation into the semantics of the **standard** predicate itself. A version of their POS is in (74):

$$(74) \quad \llbracket \text{POS} \rrbracket = \lambda G_{\langle d, et \rangle} \lambda x . \exists d [\text{standard}(G)(d) \wedge G(d)(x)]$$

This requires that there be a degree that stand in the right relation to the standard. What that relation is, precisely, depends on the adjective provided, as does whether the standard is dependent on context. Subsequently, attempts have been made to shed some light on this. Kennedy (2007) seeks to derive this effect from an economy principle. The idea is that context-dependence is inherently costly, and that there is a general linguistic preference for relying on non-contextual, lexically-provided meaning as much as possible. This would, of course, include information about scale structure. Potts (2008) suggests that this principle can actually be derived from independent principles in game theory that govern the strategies participants settle into when they interact.

These scale-structure distinctions are relevant to how adjectives behave in comparatives and other degree constructions as well. In her extensive examination of these effects, Rett (2008b,a) observes that in the equative, closed scale adjectives systematically license inferences to the positive form (as in (75)), unlike open scale adjectives (as in (76)):

- (75) a. This is as opaque as that.
 entails: This is opaque.
 b. This is as transparent as that.
 entails: This is transparent.
- (76) Floyd is as tall as Clyde.
 does not entail: Floyd is tall.

Rett’s general framework is discussed a bit more in section ???. For more on scale-structure sensitivity across syntactic categories, see Bochnak (2010), Kennedy & Levin (2008) and section ???.

For reference, (77) lists some antonyms pairs according to this aspect of scale structure. Many adjectives don’t have clear antonyms, of course. A few of the examples are less clear-cut than the others, and such cases may reflect opportunities for further refinements to the theory.

(77) ANTONYMOUS ADJECTIVES AND THEIR SCALE TYPES

<i>open</i>	<i>closed</i>	<i>upper closed</i>	<i>lower closed</i>
tall/short	empty/full	clean	dirty
heavy/light	transparent/opaque	dry	wet
high/low	open/closed	straight	bent
wide/narrow	visible/invisible	pure	impure
big/small	cooked/raw	safe	unsafe

7.3 Dimensional and non-dimensional adjectives

Bierwisch (1988, 1989) identified a scalar lexical-semantic distinction among adjectives that is manifestly related to questions of open and closed scales, but nevertheless is probably distinct. He provided a sustained argument for distinguishing between two natural classes, DIMENSIONAL and NON-DIMENSIONAL adjectives. He actually called the latter class EVALUATIVE adjectives, but I will avoid the term because it is used in a number of other ways and there is quite enough ambiguity in adjective terminology as it stands (see section ??).²⁷

Dimensional adjectives include *tall*, *heavy*, and *hot*. Non-dimensional adjectives include *stupid*, *ugly*, and *lazy*. The crucial intuition behind the distinction is that non-dimensional adjectives are ‘less clearly delimited and less systematically structured’ (Bierwisch 1988).

This intuition alone doesn’t get us very far, of course, but it correlates with a number of contrasts that are relatively clear. One of them is that dimensional adjectives come in positive-negative antonym pairs:

- (78) tall \longleftrightarrow short
heavy \longleftrightarrow light
hot \longleftrightarrow cold

Non-dimensional adjectives, on the other hand, lack a single clear antonym. Rather, they involve groups of adjectives clustered at each pole of a scale:

²⁷It’s also used to mean adjectives that simply imply some evaluative judgment such as *good* or even *unknown* (Cinque 2010), particularly in discussions of the relative order of adjectives or of implicational universals about what concepts are lexicalized as adjectives (e.g. Hetzron 1978, Laenzlinger 2000, Scott 2002, Cinque 2010). This sense of the expression is not confined to syntactic and typological literature, though (Kiefer 1978, Geuder 2000, van Rooij 2008). A closely related use characterizes subjective adjectives of the *skillful* class (Beesley 1982; see ??). Neeleman et al. (2004) and Rett (2008a,b) use it to characterize degree constructions that license inferences to the positive form. All these uses are related to Bierwisch’s sense, but none is identical to any of the others, and all things being equal a four-way (or more than four, if we include adverbs) polysemy is probably best avoided. See also sections ??, ??, ??, and ??.

- (79)
- | | | | | | | |
|---|--|---|---|---|---|---|
| { | brave
bold
courageous | } | ↔ | { | cowardly
timid
fearful | } |
| { | clever
bright
shrewd
intelligent
brilliant | } | ↔ | { | stupid
idiotic
foolish
bone-headed | } |
| { | pretty
beautiful
gorgeous
handsome | } | ↔ | { | ugly
hideous
repellant
grotesque | } |
| { | lazy
indolent
unproductive | } | ↔ | { | hard-working
industrious
workaholic | } |

One might reasonably doubt that these do in fact involve the same scale. *Shrewd* and *clever* seem to mean slightly different things, after all. But making such distinctions makes it no easier to identify a unique antonym for each of these.

Non-dimensional adjectives also have in common that they have minimal standards in the [Kennedy & McNally \(2005\)](#) sense. This wasn't how Bierwisch put it, due to his temporal precedence and lack of clairvoyance, but it seems a fair reformulation in more contemporary terms. (It may not be perfectly equivalent.) This means that in the comparative, they license inferences to the unmarked form:

- (80) NON-DIMENSIONAL
- a. Clyde is stupider than Floyd.
entails: Floyd is stupid.
 - b. Clyde is lazier than Floyd.
entails: Floyd is lazy.
 - c. Clyde is uglier than Floyd.
entails: Floyd is ugly.

As we've seen, this is not how dimensional adjectives behave:

- (81) DIMENSIONAL
- a. Clyde is taller than Floyd.
does not entail: Floyd is tall.
 - b. This is heavier than that.
does not entail: That is heavy.

Having minimal standards also ensures that unlike dimensional adjectives, non-dimensional adjectives are systematically compatible with *slightly*:

- (82) DIMENSIONAL
a. #Clyde is slightly tall.
b. #This board is slightly long.
- (83) NON-DIMENSIONAL
a. Clyde is slightly stupid.
b. Clyde is slightly lazy.

The conclusion Bierwisch draws from all this is initially startling: that non-dimensional adjectives are essentially not gradable, and therefore have no degree argument. This is immediately worrying because non-dimensional adjectives straightforwardly form comparatives, as (80) already showed, and occur with degree modifiers, as in (84):

- (84) Clyde is $\left\{ \begin{array}{l} \text{really} \\ \text{a little} \\ \text{shockingly} \end{array} \right\} \left\{ \begin{array}{l} \text{stupid} \\ \text{lazy} \\ \text{ugly} \end{array} \right\}$.

To bridge the gap, Bierwisch proposes a type shift that exploits orderings already present in the domain. Another, related possibility is that this is evidence for combining an inherent-vagueness and degree-based approach. That would make it possible to deprive non-dimensional adjectives of their degree argument without sacrificing the idea that they could indirectly become gradable. For this to be convincing, of course, it would need to be fully spelled out, and it would need to be demonstrated that this is in fact necessary to account for the differences Bierwisch observed. The minimal-standard facts alone would not be sufficient to make this case, because an alternative and independently-motivated account of those is already available, as section 7.2 showed.

7.4 *Extreme adjectives*

Certain adjectives present a puzzle to which all three of the preceding sections might be relevant. Just as many adjectives have polar antonyms, others have counterparts that, intuitively, correspond to not the opposite end of a scale but merely an extreme end of it. Among them are *gigantic*, *gorgeous*, and *fantastic* (Cruse 1986, Paradis 1997, 2001, Rett 2008b, Morzycki 2009a, 2012a).²⁸

²⁸Cruse, following Sapir (1944), called these ‘implicit superlatives’. I avoid the term because it’s not clear whether there is actually a deep connection to superlatives.

One piece of evidence that these EXTREME ADJECTIVES (henceforth EAs) are a natural class is that they occur with EXTREME DEGREE MODIFIERS (henceforth EDMs):

$$(85) \quad \text{Your shoes are } \left\{ \begin{array}{l} \text{downright} \\ \text{flat-out} \\ \text{positively} \\ \text{full-on} \end{array} \right\} \left\{ \begin{array}{l} \text{gigantic} \\ \text{gorgeous} \\ \text{fantastic} \\ \text{??big} \\ \text{??pretty} \\ \text{??OK} \end{array} \right\} !!!$$

Big, for example, is not extreme, so it resists extreme degree modification. This raises several questions: What makes an extreme adjective extreme? How this is reflected in their denotations? What makes EDMs sensitive to it? The remainder of this section will touch on some answers, distilling Morzycki (2009a, 2012a).

In addition to their ability to occur with EDMs, Cruse (1986) points out that many EAs are set apart by their ability to be ‘intensified’ via prosodic prominence:²⁹

$$(86) \quad \begin{array}{l} \text{a. That van is } \left\{ \begin{array}{l} \text{hooooooooooge} \\ \text{?biiiiiiiiiiiiiiiiig} \end{array} \right\} ! \\ \text{b. Kevin Spacey is } \left\{ \begin{array}{l} \text{fantaaaastic} \\ \text{?goooooooood} \end{array} \right\} ! \end{array}$$

In (86a), it is possible to convey greater degrees of size by pronouncing the EA *huge* with an unnaturally long vowel, and likewise for *fantastic* in (86b). This is not possible with ordinary adjectives.³⁰

Another property that distinguishes EAs, first noted by Bolinger (1967), is a resistance to comparatives and other degree constructions.

$$(87) \quad \text{??A is more excellent than B.} \quad (\text{Paradis 1997})$$

- (88) a. ?Godzilla is more gigantic than Mothra.
 b. ?Monkeys are less marvelous than ferrets.
 c. ?Everything is more scrumptious than natto.

²⁹The particular intonational contour involved in this lengthening might be crucial.

³⁰The observation that such prosodic intensification is possible, and that it is sensitive to some notion of extremeness, goes back at least to Bolinger (1972), who observed a similar contrast in nouns. This phenomenon does not seem to be simply focus, at least not in a straightforward sense—both the meaning achieved and the prosodic contour are different.

The strength of this resistance varies among speakers and among adjectives. Nevertheless, there is a class of EA comparatives whose ill-formedness is especially robust, in which an extreme and ordinary adjectives are compared:

- (89) a. #Godzilla is more gigantic than Mothra is big.
b. #Godzilla is bigger than Mothra is gigantic.

Echoing Kennedy (1997, 2001)'s term 'cross-polar anomaly', I dubbed this (less euphonically) CONFLICTING-INTENSITY ANOMALY.

There is a further distinction within the class of EAs: some are LEXICAL EXTREME ADJECTIVES, others merely CONTEXTUAL EXTREME ADJECTIVES. *Calm*, for example, can be an EA, as its compatibility with the EDM *flat-out* in (90a) attests, but this effect melts away in the context in (90b):

- (90) a. Clyde didn't panic during the earthquake—he was flat-out calm.
b. ??In his transcendental meditation class, Clyde was flat-out calm.

The crucial difference seems to be that calmness is unexpected during earthquakes, but expected during meditation. Even out-of-the-blue, expectations the rest of the sentence gives rise to can bring about this contrast:

- (91) Those $\left\{ \begin{array}{l} \text{professors} \\ ??\text{toddlers} \end{array} \right\}$ are downright illiterate.

Lexical EAs don't manifest this sensitivity. Athletes participating in the Olympics are all outstanding at their sport. But even in this context, *outstanding* seems to be an EA:

- (92) Clyde impressed everyone in the triathlon. He was downright outstanding.

The expectation that everyone is outstanding does nothing to diminish the acceptability of the EDM. Rather, what one seems to do in such examples is adjust the comparison class (or the standard of comparison) as needed. In this sense, of course, these adjectives are context-sensitive as well—but their extremeness seems to persist.

This distinction helps make sense of the comparative and degree modification data. Contextual EAs don't resist either:

- (93) a. Clyde is $\left\{ \begin{array}{l} \text{richer} \\ \text{more offensive} \\ \text{more dangerous} \end{array} \right\}$ than Floyd.

- b. Clyde is very $\left\{ \begin{array}{l} \text{rich} \\ \text{offensive} \\ \text{dangerous} \end{array} \right\}$.

This also correlates with another difference: lexical EAs often have ‘neutral’ counterparts to which they license entailments (*gigantic* entails *big*), but contextual EAs do not.

The account in [Morzycki \(2012a\)](#) is built on an analogy to quantification generally. When we assert that *Everyone left*, we don’t actually commit to the entire population of the globe having (improbably) left. Rather, we confine the domain of the quantifier to a smaller set of individuals ([Westerståhl 1985](#), [von Stechow 1994](#) among many others). This seems to be how natural language quantification works in general. That being the case, we should expect the existential quantifier in POS to be similarly restricted. This, in turn, means that in any given context of use, we don’t attend to an entire degree scale. Rather, we attend only to salient degrees, which constitute only a part of the scale, which I called the PERSPECTIVE SCALE. With other forms of quantification, there are morphemes that signal we should extend the domain to include individuals we might not otherwise have, such as *any* or *ever* ([Kadmon & Landman 1993](#)). EAs can be viewed as analogous. They lexically encode that we should consider a degree outside of the perspective scale. Concretely, one can represent the set of salient degrees (the perspective scale) with a contextually-supplied variable, *C*, leading to denotations like those in (94) and (95):

(94) ORDINARY ADJECTIVE

- a. $\llbracket \text{big}_C \rrbracket = \lambda x \lambda d . d \in C \wedge \mathbf{big}(d)(x)$
 b. $\llbracket \text{POS big}_C \rrbracket = \lambda x . \exists d [d \in C \wedge \mathbf{big}(d)(x) \wedge d > \mathbf{standard}(\mathbf{big})]$

(95) EXTREME ADJECTIVE

- a. $\llbracket \text{gigantic}_C \rrbracket = \lambda x \lambda d . d > \mathbf{max}(C) \wedge \mathbf{big}(d)(x)$
 b. $\llbracket \text{POS gigantic}_C \rrbracket = \lambda x . \exists d \left[\begin{array}{l} d > \mathbf{max}(C) \wedge \mathbf{big}(d)(x) \wedge \\ d > \mathbf{standard}(\mathbf{big}) \end{array} \right]$

The ordinary adjective *big* is interpreted exactly as we’d expect, with the additional twist that the degree quantifier now has a contextual domain restriction. The EA *gigantic*, on the other hand, lexically encodes that a degree must exceed (the maximum degree of) the set of salient degrees *C*, thereby capturing the sense that to be *gigantic* is not merely to be very big, but to be big to degrees that exceed contextual expectations. This mechanism makes possible an account of how lexical EAs behave in comparatives and equatives as well, and might provide a way of thinking about imprecision via

perspective scale granularity (along lines suggested by [Sauerland & Stateva 2007](#)).

What EDMs do is manipulate perspective scales. The general idea is this: because EAs differ from ordinary adjectives in the degrees that satisfy them, EDMs can impose the requirement that makes them compatible only with EAs by simply having certain presuppositions about the scale structure of the adjective they combine with.

7.5 *Gradable modal adjectives*

There is another type of gradation that has long attracted the attention of semanticists: the kind associated with modals, including most prominently modal auxiliaries. In evaluating claims involving modals, it's frequently necessary to consider various non-optimal circumstances. It's true, for example, that the law is that if you murder Floyd, you must go to jail. But the law also says that you must not murder Floyd in the first place. A world in which Floyd murdered someone and went to jail accords better with the law than one in which he did so and got away, but neither accords with it fully. So when evaluating what the law says must happen, the worlds that we have to take into account can't be only the ones in which all laws are fully met. Some of them need to be worlds that fail to fully accord with the law. The classical analysis of [Kratzer \(1981\)](#) therefore has as a crucial ingredient an ordering among worlds ([Lewis 1973](#)) according to how fully they accord with a set of requirements (an ORDERING SOURCE) such as the law. This ordering relation on its own is already suggestive of a potential connections to degree orderings, but there is in fact more to it. Kratzer goes on to rigorously define notions like 'slight possibility' and 'human possibility' (expressed in ordinary language as *a good possibility*), and even 'comparative possibility' (the analogue of *it's more likely than*). So, as [Portner \(2009\)](#) points out, these questions of modality land us directly on the turf of degree semantics.

The question, then, is what to conclude from this. If these modal tools are sufficient to account for gradability in general, we should try to use those, since they also account for an independent set of facts. If not, the situation becomes more complicated. Of course, it is, in fact, more complicated. One difficulty a theory based on ordering worlds has is with sentences like (96):

- (96) a. It's twice as likely that Godzilla will eat Mothra.
b. There is a 50% probability that Godzilla will eat Mothra.

The problem is making sense of apparently numerical notions like '50%' and 'twice' in a theory that has no way to represent them. [Swanson \(2006\)](#), [Villalta \(2007\)](#), [Yalcin \(2007\)](#), [Portner \(2009\)](#), [Lassiter \(2010, 2011b,a\)](#),

Klecha (2012, 2013) have in various ways taken up the challenge of reconciling or combining the standard way of thinking about gradation and modality with the kind of data for which degree semantics is designed.

The most natural place to make this connection is, naturally enough, gradable modal adjectives like *likely*, *possible*, and *certain*. One satisfyingly direct move, explored from a linguistic perspective most extensively by Lassiter, is to treat these adjectives as involving direct measures of probability.³¹ This would, of course, represent a major departure from the classical treatment of modals—and if that departure is warranted for these adjectives, it might suggest that the account of other modals should undergo a similar shift. So a great deal is at stake. Klecha (2012) seeks to chart a middle course that preserves aspects of the classical analysis of modals on a degree semantics for these adjectives. Part of the interest of this area is that it lies at the intersection of two well-studied areas of the grammar, with consequences for both and opportunities to ask questions that relate the two. One can, for example, examine the scale structure of gradable modals and their interaction with degree modifiers—and indeed, Lassiter (2011a) and Klecha (2012) both do.

7.6 On scales and categories

Chapter ?? ended on a slightly pessimistic note about whether one could find straightforward cross-linguistic semantic correlates of being an adjective. This section returns to that question in a more optimistic spirit.

It's certainly true that not all adjectives are gradable and not all gradable categories are adjectives. As we've seen, vagueness is ubiquitous, and one of the principal ways of thinking about it—the inherent vagueness approach—makes no deep compositional distinction between adjectives and other predicative categories. Yet this draws attention to an important contrasting property of the degree-based approach. Foundational to it is the idea that gradable adjectives have a different type from other predicates. That type might be $\langle d, et \rangle$ or $\langle e, dt \rangle$ or $\langle e, d \rangle$, but one way or another, a degree is directly involved. Might this type difference provide a way of matching a syntactic category with a type? Types are, after all, the building blocks of a kind of parallel syntax running inside the semantics, with its own conditions on well-formedness and legal modes of combination.

I suspect the answer is no, but I wouldn't bet on it. Kennedy & Levin (2008) and Piñón (2008), for example, invoke degrees in the semantics of verbs, and Morzycki (2005, 2009b) does so for nouns. On the other hand,

³¹The idea of introducing probability measures into the grammar is older, dating to at least Kamp (1975), and in some form as far as Black (1937). Thanks to Daniel Lassiter (p.c.) for pointing this out.

Rett (2011) makes the case against verbal degree arguments, and Morzycki (2012b) backpedals about just how big a role degrees play in nouns too. Perhaps one way of splitting the difference is to suppose that outside of adjectives, degrees are involved but never as arguments. Alternatively, it may be that they are involved, but only in certain lexically exceptional cases—that is, that only the most adjective-like nouns and verbs have degree arguments.³²

This isn't a question of degree arguments alone, however. There do seem to be generalizations about the scalar properties of syntactic categories that draw distinctions independently of any particular theory of gradability. It is an old observation that adjectives frequently involve a single dimension or 'quality', whereas nouns involve many (see Hamann 1991 and references there). To be *blue*, for example, one must have a single, irreducible quality: blueness. To be a chair, one must have many qualities, and we could enumerate at least some of them. Because we couldn't make a full list of necessary and sufficient conditions, though, it's probably better to view noun meanings in terms of prototypes: a chair must be sufficiently like a prototypical chair (Rosch 1973, Osherson & Smith 1981, 1997, Kamp & Partee 1995). That would make nouns even more unlike adjectives.

A skeptic might object that comparing chairs and blue things is a way of rigging the game by picking extreme examples—of course chairs are more complicated than blue things, but that's an insight into home furnishings, not language. But then consider the canonical example of vagueness we began with, which involved a noun: *heap*. There are at least two ways of failing to be a heap, or of becoming only a borderline case of a heap: by having too few grains, as in the sorites paradox, or by being too flat. An agglomeration of many grains of sand that is perfectly spread out, one grain deep, is definitely not a heap. So there are two qualities or dimensions here, independently discernible. In contrast, there is only one way of failing to be tall: by being too short. Again, one might object that our choice of adjectives rigged the game as well. There are more complicated adjectives. Bierwisch's non-dimensional adjectives may be an example. Indeed, Sassoon (2013a,b) explicitly argues that many adjectives involve multiple dimensions (but nevertheless handle those dimensions differently from nouns).

So where does this leave us? Perhaps we can't find a single, simple, rigorous, and crosslinguistic semantic definition of adjectives. But there is certainly room for identifying semantic correlates of syntactic category, particularly with respect to the connection between adjectives and scales. At a minimum, we can say with some confidence that there are certain semantic characteristics that, if not perfectly correlated with adjectivehood, are at least

³²We will return to the issue of non-adjectival gradability in section ??.

unmistakably adjectivey.

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